1. $f(x) = \cos(5x)$

$f'(x) = \boxed{(-\sin(5x)) \cdot 5} = -5\sin 5x$

2. $f(x) = \ln(\tan x)$

$f'(x) = \boxed{\frac{1}{\tan x} \cdot \sec^2 x} = \frac{\sec^2 x}{\tan x} = \frac{\frac{1}{\cos^2 x}}{\frac{\sin x}{\cos x}} = \frac{1}{\cos^2 x} \cdot \frac{\cancel{\cos x}}{\sin x} = \frac{1}{\sin x \cos x} = \csc x \sec x$

3. $f(x) = e^{\arctan 2x}$

$f'(x) = \boxed{e^{\arctan 2x} \cdot \frac{1}{1+(2x)^2} \cdot 2}$

4. $f(x) = \sqrt[3]{\sec x} = (\sec x)^{1/3}$

$f'(x) = \boxed{\frac{1}{3}(\sec x)^{-2/3} \cdot \sec x \tan x}$

5. $f(x) = -4\sin(3e^x)$

$f'(x) = \boxed{-4\cos(3e^x) \cdot (3e^x)}$

1. $f(x) = (x^2)(\sin x)$

$f'(x) = x^2\cos x + 2x\sin x$

2. $f(x) = \frac{\tan x}{e^x}$

$f'(x) = \frac{e^x \sec^2 x - e^x \tan x}{(e^x)^2}$

3. $f(x) = \ln x - \sec x$

$f'(x) = \frac{1}{x} - \sec x \tan x$

4. $f(x) = 3^x + \log_4 x$

$f'(x) = 3^x \ln 3 + \frac{1}{x \ln 4}$

5. $f(x) = \sqrt{\cos x} = (\cos x)^{1/2}$

$f'(x) = \frac{1}{2}(\cos x)^{-1/2} \cdot (-\sin x)$

6. $f(x) = 2\cot^3 x = 2(\cot x)^3$

$f'(x) = 6(\cot x)^2 \cdot (-\csc^2 x)$

7. $f(x) = 5^{\csc x}$

$f'(x) = 5^{\csc x} \cdot \ln 5 \, (-\csc x \cot x)$

8. $f(x) = \sin(e^{2x})$

$f'(x) = \cos(e^{2x}) \cdot e^{2x} \cdot 2$

$$f(x) = \frac{\csc(\ln(x^2))}{37x + \sin x}$$

$(\sec x)' = \sec x \tan x$

$[\sec[f(x)]]' = \sec(f(x))\tan(f(x)) \cdot f'(x)$

$$f'(x) = \frac{\left[(37x + \sin x)\cdot\left(-\csc(\ln(x^2))\cot(\ln(x^2))\right)\cdot\frac{1}{x^2}\cdot 2x\right] - \left[\csc(\ln(x^2))\cdot(37 + \cos x)\right]}{(37x + \sin x)^2}$$

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

$$\left[2^{f(x)}\right]' = 2^{f(x)} \cdot \ln 2 \cdot f'(x)$$

$$\left[\left[f(x)\right]^2\right]' = 2f(x) \cdot f'(x)$$

$$\left[2^{\left([f(x)]^2\right)}\right]' = 2^{[f(x)]^2} \cdot \ln 2 \cdot 2 \cdot f(x) \cdot f'(x)$$

Derivatives - must know ALL rules except inverse trig functions (arcsin, etc), but still must be able to apply those given the formulas.

$[x^n]' = nx^{n-1}$

$[cf(x)]' = c\,f'(x)$

$[f(x) \pm g(x)]' = f'(x) \pm g'(x)$

$[f(x)g(x)]' = f'(x)g(x) + g'(x)f(x)$

$\left[\dfrac{f(x)}{g(x)}\right]' = \dfrac{g(x)f'(x) - g'(x)f(x)}{[g(x)]^2}$

$[f(g(x))]' = f'(g(x)) \cdot g'(x)$

$[e^x]' = e^x$

$[a^x]' = a^x \ln a$

$[\ln x]' = \dfrac{1}{x}$

$[\log_a x]' = \dfrac{1}{x \ln a}$

$[\sin x]' = \cos x$

$[\cos x]' = -\sin x$

$[\tan x]' = \sec^2 x$

$[\cot x]' = -\csc^2 x$

$[\sec x]' = \sec x \tan x$

$[\csc x]' = -\csc x \cot x$

$[\arcsin x]' = \dfrac{1}{\sqrt{1-x^2}}$

$[\arctan x]' = \dfrac{1}{1+x^2}$

$[\operatorname{arcsec} x]' = \dfrac{1}{|x|\sqrt{x^2-1}}$

$[\arccos x]' = \dfrac{-1}{\sqrt{1-x^2}}$

$[\operatorname{arccot} x]' = \dfrac{-1}{1+x^2}$

$[\operatorname{arccsc} x]' = \dfrac{-1}{|x|\sqrt{x^2-1}}$

Second, third, ..., nth derivatives

$$f''(x) = [f'(x)]'$$

$$f(x) = \sin x$$
$$f'(x) = \cos x$$
$$f''(x) = -\sin x$$

$$f(x) = 59x^{23} - 4x^5 + 2$$
$$f^{(25)}(x) = 0$$

Derivative using limit definition

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x) = 3x^3 - 2x + 5$$

$$f'(x) = \lim_{h\to 0} \frac{3(x+h)^3 - 2(x+h) + 5 - (3x^3 - 2x + 5)}{h}$$

$$= \lim_{h\to 0} \frac{3(x^3 + 3x^2h + 3xh^2 + h^3) - 2x - 2h + 5 - 3x^3 + 2x - 5}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{3x^3} + 9x^2h + 9xh^2 + 3h^3 - \cancel{2x} - 2h + \cancel{5} - \cancel{3x^3} + \cancel{2x} - \cancel{5}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(9x^2 + 9xh + 3h^2 - 2)}{\cancel{h}} = \boxed{9x^2 - 2}$$

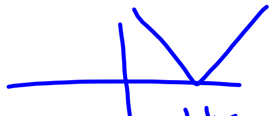
Alternate limit definition to show derivative does not exist at a given point

$$f'(c) = \lim_{x\to c} \frac{f(x) - f(c)}{x - c}$$

$(c, f(c))$

1. $f(x) = |x-3|$ @ $(3, 0)$

$f(x) = \sqrt[3]{x-1}$ @ $(1, 0)$



1. $\lim_{x\to 3^-} \frac{|x-3| - 0}{x-3} = \lim_{x\to 3^-} \frac{|x-3|}{x-3} = -1$

$\lim_{x\to 3^+} \frac{|x-3| - 0}{x-3} = \lim_{x\to 3^+} \frac{|x-3|}{x-3} = 1$

left- & right-hand limits different, hence limit in general & derivative defined by that limit do not exist

2. $\lim_{x\to 1} \frac{\sqrt[3]{x-1} - 0}{x-1} = \lim_{x\to 1} \frac{(x-1)^{1/3}}{(x-1)^1} = \lim_{x\to 1} \frac{1}{(x-1)^{2/3}} = \infty$

infinite slope → vertical tangent line

derivative does not exist

Tangent line slope and/or equation for a function at a given point

$f(x)$ @ $(c, f(c))$

$m = f'(c)$

$y - f(c) = f'(c)[x - c]$

Instantaneous and/or average rate of change

for $f(x)$

inst. @ $x = c \Rightarrow f'(c)$

avg. as x goes from a to b

$$\frac{f(b) - f(a)}{b - a}$$

$y = mx + d$ $f(x) = ax^2 + bx + c$

when a line is tangent to a function, they touch (intersect) @ that point

$\boxed{mx + d = ax^2 + bx + c}$

derivative of function yields slope of tangent line

$\boxed{m = 2ax + b}$

solve the system of equations

Homework for Test #2 on Derivatives

HW #4 (submitted Fri 12/05)

- 2.1 #1-23 odd Find the derivative by the limit process
- #29-32 all find the equation of the tangent line
- #61-69 odd Use the alternate form to find the derivative
- #71-79 odd Describe x-values where the function is differentiable (given graph)
- 2.2 #3-51 odd Find the derivative using the basic derivative rules
- #91-94 all; 101, 102 use the derivative to solve rate of change word problems

HW #5 (submitted Mon 12/15)

- 2.3 #1-53 odd, 63-69 odd, 75-81 all, 83-91 odd, 109-115 all — Product and quotient rules
- 2.4 #7-33 odd, #47-81 odd — Chain rule
- 5.1 #45-61, 71 — Logarithmic functions
- 5.4 #39-57 — Exponential functions
- 5.5 #41-55 — Log and exp functions with other bases
- 5.8 #41-59 — Inverse trig functions

HW #6 (due test day) - Practice problem handout (Prac Probs & Old Test from web site)

TEST #2 - WEDNESDAY? (OR WED/FRI)