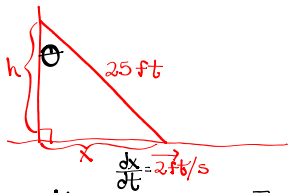
27.



(a) $\frac{dh}{dt} = ?$ when $x = 7$ ft

$h = \sqrt{25^2 - x^2}$

$h = \sqrt{25^2 - 7^2}$

$= 24$

$x^2 + h^2 = 25^2$

$2x\frac{dx}{dt} + 2h\frac{dh}{dt} = 0$

$\frac{dh}{dt} = \frac{-2x\cdot\frac{dx}{dt}}{2h} = \frac{-2(7)(2)}{2(24)}$

$= \boxed{\frac{-7}{12}\ \text{ft/s}}$

(b) $\frac{dA}{dt} = ?$ when $x = 7$ ft

$A = \frac{1}{2}xh = \frac{1}{2}x\sqrt{25^2 - x^2} = \frac{1}{2}x\left(25^2 - x^2\right)^{1/2}$

$\frac{dA}{dt} = \frac{1}{2}\frac{dx}{dt}\sqrt{25^2 - x^2} + \left(\frac{1}{2}x\right)\cdot\frac{1}{2}\left(25^2 - x^2\right)^{-1/2}\cdot\left(-2x\cdot\frac{dx}{dt}\right)$

(plug in 2 for dx/dt & 7 for x)

(c) $\frac{d\theta}{dt} = ?$ when $x = 7$ ft

$\sin\theta = \frac{x}{25} = \frac{1}{25}x$

$\cos\theta\cdot\frac{d\theta}{dt} = \frac{1}{25}\cdot\frac{dx}{dt}$

$\frac{d\theta}{dt} = \frac{\frac{1}{25}\frac{dx}{dt}}{\cos\theta} = \frac{\frac{1}{25}(2)}{24/25} = \frac{1}{12}$ rad/s

when $x = 7$,

$h = 24$ (by Pyth. Th.)

$\Rightarrow \cos\theta = \frac{h}{25} = \frac{24}{25}$

1. identify knowns & unknowns
2. find formula(s) to relate them
   (big formula (volume, etc)
   & little formula (relating r & h or x & h))
3. Simplify big formula
   (using little formula & algebra)
4. take derivative (w.r.t. time)
5. Solve for unknown &
   plug in known constants
   (when h = 3, etc.)

Find the derivative of f with respect to x.

$$f(x) = 5\sin^2\left(\sqrt{3\csc(7x^2 - 2x)}\right)$$

$$= 5\left[\sin\left(\left[3\csc(7x^2-2x)\right]^{1/2}\right)\right]^2$$

$$f'(x) = 10\left[\sin\left(\left[3\csc(7x^2-2x)\right]^{1/2}\right)\right] \cdot$$
$$\cdot \cos\left(\left[3\csc(7x^2-2x)\right]^{1/2}\right) \cdot$$
$$\cdot \tfrac{1}{2}\left[3\csc(7x^2-2x)\right]^{-1/2} \cdot$$
$$\cdot (-3\csc(7x^2-2x)\cot(7x^2-2x)) \cdot$$
$$\cdot (14x-2)$$

$f(x) = -3x\tan x$

a. Find $f'(x)$. find $\frac{d}{dx}[f(x)]$

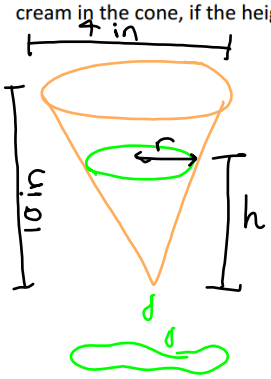
b. Find $f''(x)$. find $\frac{d^2}{dx^2}[f(x)]$

(a) $f'(x) = (-3)\tan x + (-3x)(\sec^2 x)$

$= -3\tan x - 3x(\sec x)^2$

(b) $f''(x) = -3\sec^2 x - 3\sec^2 x + (-3x)(2\sec x \cdot \sec x\tan x)$

1. A jumbo waffle cone from Sarah's Tasty Ice Cream Shoppe is 10 inches tall and has a 4 inch diameter at the top of the cone. Yesterday, my cone had a leak! Instead of eating it super fast, I decided to compare the rate of change of volume of ice cream to the rate of change of height of ice cream in the cone. How fast is the ice cream leaking out (in cubic inches per minute) when there are 5 inches of ice cream in the cone, if the height of ice cream in the cone is changing at a rate of 1 inch every 5 minutes?



$\frac{dV}{dt} = ?$ when $h = 5$ in ; $\frac{dh}{dt} = \frac{1 \text{ in}}{5 \text{ min}}$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \left(\frac{h}{5}\right)^2 \cdot h$

$\frac{r}{h} = \frac{2}{10}$

$r = \frac{h}{5}$

$V = \frac{\pi}{75} h^3$

$\frac{dV}{dt} = \frac{\pi}{75}(3h^2) \cdot \frac{dh}{dt}$

$= \frac{\pi}{25}(5 \text{ in})^2 \cdot \frac{1 \text{ in}}{5 \text{ min}}$

$= \boxed{\frac{\pi}{5} \frac{\text{in}^3}{\text{min}}}$

2. $x^3 + y^2 = 10$

a. Find y' in terms of x and y.

b. Find y'' in terms of x and y.

(a) $\frac{d}{dx}[x^3 + y^2] = \frac{d}{dx}[10]$

$3x^2 + 2yy' = 0$

$\boxed{y' = \frac{-3x^2}{2y}}$

(b) $y'' = \frac{(2y)(-6x) - (-3x^2)(2y')}{(2y)^2}$

$y'' = \frac{-12xy + 6x^2\left(\frac{-3x^2}{2y}\right)}{4y^2}$

$y'' = \frac{-12xy - \frac{9x^4}{y}}{4y^2}$

$= \frac{-\frac{12xy^2}{y} - \frac{9x^4}{y}}{4y^2}$

$\boxed{y'' = \frac{-12xy^2 - 9x^4}{4y^3}}$ (book answer)

HW #7 (due Fri, 9 Jan)
2.5 # 1-39 odd; 43, 47 - Implicit Differentiation
2.6 # 15-23 odd - Related Rates
2.6 # 25, 27, 35 - Related Rates (more challenging problems)

HW #8 (due Fri, 16 Jan)
Test 3 Practice Problems Handout (NOT ON WEB)

Quiz #5 - Fri, 16 Jan (related rates)
Test #3 - Tues, 20 Jan (implicit differentiation, related rates, & review problems)

HW #9
3.1 # 17-31 odd - Absolute Extrema on an Interval
3.2 # 7-19 odd - Rolle's Theorem
3.2 # 31-37 odd - Mean Value Theorem
3.3 # 11-31 odd - Increasing, Decreasing, and Relative Extrema
Quiz #6?
HW #10
3.4 #11-25 odd - Inflection Points and Concavity
3.5 #15-31 odd - Limits at Infinity