HW #9 (due Fri, 23 Jan)
3.1 # 17-31 odd - Absolute Extrema on an Interval
3.2 # 7-19 odd - Rolle's Theorem
3.2 # 31-37 odd - Mean Value Theorem
Quiz #6 - Mon, 26 Jan

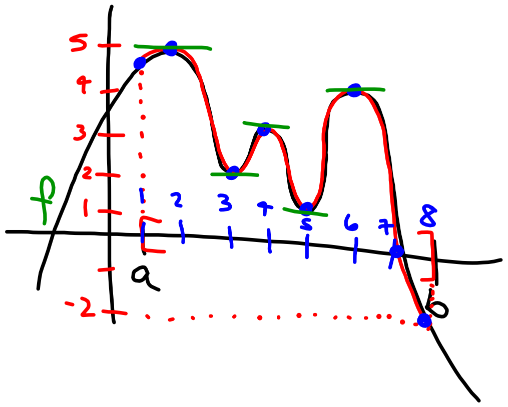
HW #10 (due Fri, 30 Jan)
3.3 # 11-31 odd - Increasing, Decreasing, and Relative Extrema
3.4 #11-25 odd - Inflection Points and Concavity
3.5 #15-31 odd - Limits at Infinity
7.7 #11-35 odd - l'Hopital's Rule
Quiz #7 - Fri, 30 Jan

HW #11 (due Wed, 4 Feb)
7.7 #37-53 odd - l'Hopital's Rule with logs
3.7 #3,5,17,23,29 - Optimization
Test 4 - Wed, 4 Feb

Final Exam - Thurs, 12 Feb 1:00-3:00pm

3.1 Extrema on an Interval

→ maxima & minima
→ relative & absolute



relative minima:
(3,2), (5,1)

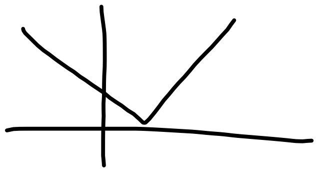
relative maxima:
(2,5), (4,3), (6,4)

absolute maximum:
5 @ (2,5)

absolute minimum:
-2 @ (8,-2)

f(x) has a relative maximum or minimum when f'(x) = 0. or

f'(x) is undefined.

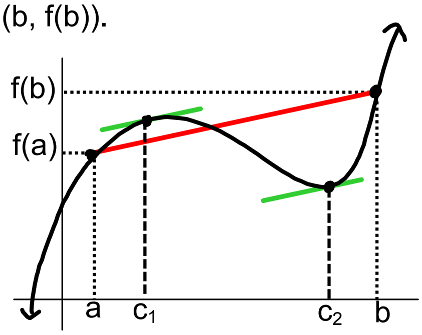


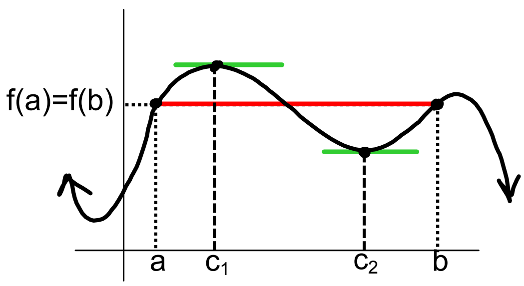
We call such x-values

Critical #'s of f.

3.2 Rolle's Theorem & The Mean Value Theorem

The Mean Value Thereom (MVT) states: If f is continuous on [a,b] and differentiable on (a,b), then there exists at least one c in (a,b) such that the slope of the tangent line at c is equal to the slope of the secant line through (a, f(a)) and (b, f(b)).





Rolle's Theorem is a special case of the MVT where f(a)=f(b), (and hence involving horizontal secant/tangent lines)

MVT: If f is continuous on $[a,b]$ and differentiable on (a,b), then there exists at least one $c \in (a,b)$ such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

slope of tangent line to f @ c

slope of secant line through $(a, f(a))$ & $(b, f(b))$

~~MVT~~ Rolle's Thm: If f is continuous on $[a,b]$ and differentiable on (a,b) and $f(a)=f(b)$, then there exists at least one $c \in (a,b)$ such that

$$f'(c) = \cancel{\frac{f(b)-f(a)}{b-a}} \; 0$$

slope of tangent line to f @ c

slope of secant line through $(a, f(a))$ & $(b, f(b))$

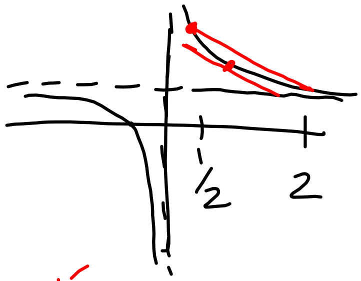
f is continuous at c if $\lim_{x\to c} f(x) = f(c)$

f is continuous on closed interval $[a, b]$ If f is continuous for all $x \in (a,b)$ and

$\lim_{x\to a^-} f(x) = f(a)$ & $\lim_{x\to b^+} f(x) = f(b)$

Can the Mean Value Theorem be applied?
If so, find all guaranteed values of c in (a,b).

34. $f(x) = \dfrac{x+1}{x}$, $\left[\frac{1}{2}, 2\right]$
a , b



Steps to solve MVT problems:
1. Is f continuous on [a,b]? yes
2. Is f differentiable on (a,b)? yes } MVT applies
3. Find (f(b)-f(a))/(b-a) = −1
4. Find f'(x)
5. Set #3&4 equal, solve for x
6. Solution is the values of x from #5 that lie in (a,b)

$$\frac{\frac{2+1}{2} - \frac{\frac{1}{2}+1}{\frac{1}{2}}}{2-\frac{1}{2}} = \frac{\frac{3}{2}-\frac{6}{2}}{\frac{3}{2}} = -1$$

$$f(x) = \frac{x+1}{x}$$

$$f'(x) = \frac{x(1)-(x+1)(1)}{x^2} = -\frac{1}{x^2}$$

$$\frac{-1}{c^2} = -1$$

$$-1 = -c^2$$

$$1 = c^2$$

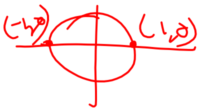
$$c = \pm 1$$

$$\boxed{c = 1}$$

38. $f(x) = 2\sin x + \sin 2x \;,\; [0, \pi]$

f is cts. on $[0,\pi]$? yes
f is diff on $(0,\pi)$? yes
} MVT applies

$$\frac{f(b)-f(a)}{b-a} = \frac{2\sin\pi + \sin 2\pi - (2\sin 0 + \sin 2(0))}{\pi - 0} = 0$$



$f'(x) = 2\cos x + 2\cos 2x$

$2\cos x + 2\underbrace{\cos 2x} = 0$

$2\cos x + 2(2\cos^2 x - 1) = 0$

$4\cos^2 x + 2\cos x - 2 = 0$

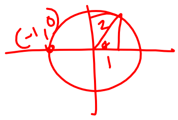
$2\cos^2 x + \cos x - 1 = 0$

$2u^2 + u - 1 = 0$
$(2u-1)(u+1) = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$2\cos x - 1 = 0$
$\cos x = 1/2$
$\boxed{x = \pi/3}$

~~$\cos x + 1 = 0$~~
~~$\cos x = -1$~~



32. $f(x) = x(x^2 - x - 2) \quad [-1, 1]$

f is both continuous & differentiable on $[-1, 1]$ on $(-1,1)$ $\Rightarrow$ MVT applies

$$\frac{f(1)-f(-1)}{1-(-1)} = \frac{1(1-1-2)-(-1)(1+1-2)}{2} = \frac{-2+0}{2} = -1$$

$f(x) = x^3 - x^2 - 2x$
$f'(x) = 3x^2 - 2x - 2$

$3x^2 - 2x - 2 = -1$
$3x^2 - 2x - 1 = 0$
$(3x+1)(x-1) = 0$

$x = -1/3$ ~~$x = 1$~~ $\leftarrow$ not on closed interval

3.3-3.4 Increasing, Decreasing, Concavity, and the 1st and 2nd Derivative Tests

What do *f′* and *f″* tell us about *f*?

Recall that *f′* is the rate of change or slope of *f*,
f″ is the slope or rate of change of *f′*.

f'	f
+	↗ increasing
−	↘ decreasing

f''	f'	f
+	↗ increasing	concave up
−	↘ decreasing	concave down

f′(x)=0 when f has a relative maximum or minimum.
These x-values (and those where f'(x) is undefined) are called critical numbers.

f″(x)=0 when f changes concavity.
The points where concavity changes are called inflection points.