# Diff Cal - 3.3-3.4 - Increasing/Decreasing, Extrema, Concavity & Inflection Points

HW #9 (due Fri, 23 Jan)
3.1 # 17-31 odd - Absolute Extrema on an Interval
3.2 # 7-19 odd - Rolle's Theorem
3.2 # 31-37 odd - Mean Value Theorem
Quiz #6 - Mon, 26 Jan

**HW #10 (due Fri, 30 Jan)**
**3.3 # 11-31 odd - Increasing, Decreasing, and Relative Extrema**
**3.4 #11-25 odd - Inflection Points and Concavity**
**3.5 #15-31 odd - Limits at Infinity** ←
**7.7 #11-35 odd - l'Hopital's Rule** ←
**Quiz #7 - Fri, 30 Jan**

HW #11 (due Wed, 4 Feb)
7.7 #37-53 odd - l'Hopital's Rule with logs
3.7 #3,5,17,23,29 - Optimization
Test 4 - Wed, 4 Feb

Final Exam - Thurs, 12 Feb 1:00-3:00pm

1. Is f continuous on $[-3,2]$?
Is f differentiable on $(-3,2)$? } Yes
Is $f(-3)=f(2)$? Yes $(f(-3)=0=f(2))$

$\Rightarrow$ Rolle's Thm applies

$f(x)=x^2+x-6$

$f'(x)=2x+1$

$2x+1=0$

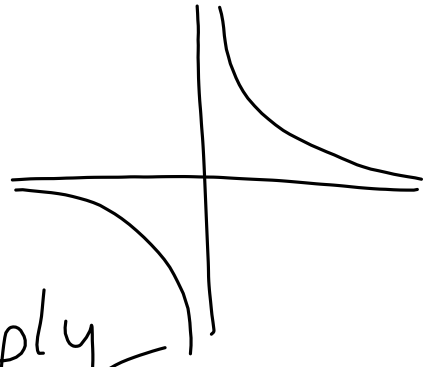
$2x=-1$

$\boxed{x=-1/2}$

2. $f(x) = \frac{1}{x}$ , $[-1, 4]$

$f$ has a discontinuity @ $0 \in [-1, 4]$

$\Rightarrow$ MVT does not apply



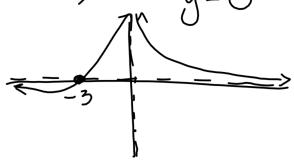
3.3

30. $f(x) = \frac{x+3}{x^2}$ $\frac{-2\cdot 3}{3\cdot 3\cdot 3\cdot 3}$

V.A. @ $x = 0$

real zero/x-int @ $x = -3$

end behavior: $\frac{x}{x^2} = \frac{1}{x}$ H.A. @ $y = 0$




$f'(x) = \frac{x^2(1) - (x+3)(2x)}{(x^2)^2}$

$= \frac{x^2 - 2x^2 - 6x}{x^4} = \frac{-x^2 - 6x}{x^4}$

$= \frac{x(-x-6)}{x^4} = \frac{-(x+6)}{x^3}$

$\boxed{\text{critical \#'s: } -6, 0}$

$f'(-7)$ $f'(-1)$ $f'(1)$

$-$ $-6$ $+$ $0$ $-$

$\boxed{\begin{array}{l} f \text{ is decreasing on } (-\infty, -6) \cup (0, \infty) \\ \text{\& Increasing on } (-6, 0) \\ f \text{ has relative/absolute min @ } (-6, -\frac{1}{12}) \end{array}}$

$f'(x) = \frac{-x-6}{x^3}$

$f''(x) = \frac{(x^3)(-1) - (-x-6)(3x^2)}{x^6} = \frac{x^2[x(-1) - (-x-6)\cdot 3]}{x^6} = \frac{-x + 3x + 18}{x^4}$

$= \frac{2x+18}{x^4}$

$2x + 18 = 0$

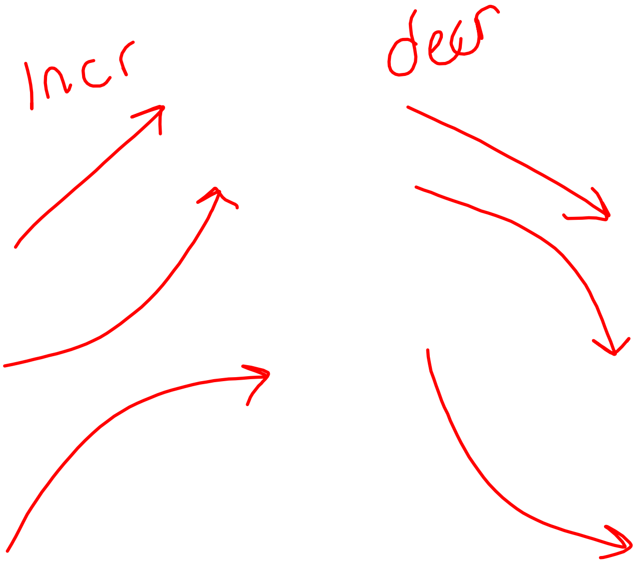
$2x = -18$

$x = -9$

$f''(-10)$ $f''(-1)$ $f''(1)$

$-$ $-9$ $+$ $0$ $+$

$\boxed{\begin{array}{l} f \text{ is concave down } (-\infty, -9) \\ \text{\& concave up on } (-9, 0) \cup (0, \infty) \\ f \text{ has inflection point @ } (-9, -\frac{2}{27}) \end{array}}$



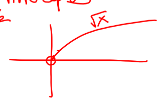

3.4 #20

$$f(x) = \frac{x+1}{\sqrt{x}}$$

domain: $(0, \infty)$

no x- or y- intercepts

$\frac{x}{\sqrt{x}} = \frac{x^1}{x^{1/2}} = x^{1/2}$




$$f'(x) = \frac{\sqrt{x}(1) - (x+1)\cdot\left(\frac{-1}{\sqrt{x}}\right)}{x}$$

$$= \frac{\sqrt{x} + \frac{x}{\sqrt{x}} + \frac{1}{2\sqrt{x}}}{x} = \frac{x^{1/2} + x^{1/2} + \frac{1}{2x^{1/2}}}{x^1}$$

$$= \frac{2x^{1/2} + \frac{1}{2x^{1/2}}}{x^1} = \frac{2}{x^{1/2}}\frac{2x}{2x} + \frac{1}{2x^{3/2}} = \frac{4x+1}{2x^{3/2}}$$

critical #'s: $-\frac{1}{4}$, 0 (neither in domain!

$f'$(positive #)

0 +

$\boxed{\text{f is increasing on } (0, \infty) \text{ no extrema}}$

$$f'(x) = \frac{4x+1}{2x^{3/2}}$$

$x^{1/2}[2x^{2/2}(4) - (4x+1)(3)]$

$$f''(x) = \frac{2x^{3/2}(4) - (4x+1)(3x^{1/2})}{4x^3}$$

$$= \frac{8x - 12x - 3}{4x^{5/2}} = \frac{-4x-3}{4x^{5/2}}$$

$-4x - 3 = 0$

$-4x = 3$

$x = -3/4$ ← not in domain

$f''$(positive #)

0 −

$\boxed{\text{f is concave down on } (0, \infty)}$

$\boxed{\text{\& has no inflection points}}$