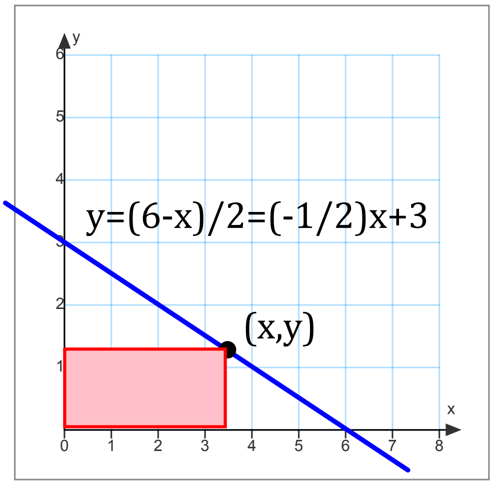
24. A rectangle is bounded by the x- and y-axes and the graph of y=(6-x)/2. What length and width should the rectangle have so that its area is a maximum?



$$A(x) = x\left(\frac{6-x}{2}\right)$$

$$= x\left(-\frac{1}{2}x + 3\right)$$

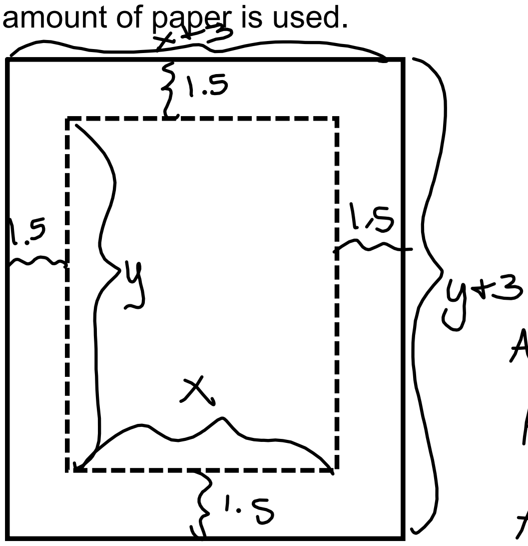
$$A(x) = -\frac{1}{2}x^2 + 3x$$

$$A'(x) = -x + 3 \qquad -x+3=0 \quad x=3$$

max area occurs when $X=3$ (width) &

$$y = \frac{6-3}{2} = \frac{3}{2} \text{ (height)}$$

30. A rectangular page is to contain 36 square inches of print. The margins on each side are to be 1.5 inches. Find the dimensions of the page such that the least amount of paper is used.



minimize area of paper

$$xy = 36 \Rightarrow y = \frac{36}{x}$$

$$A(x) = (x+3)\left(\frac{36}{x}+3\right)$$

$$A(x) = 36 + 3x + \frac{3(36)}{x} + 9$$

$$A(x) = 45 + 3x + 3(36)x^{-1}$$

$$A'(x) = 3 - \frac{3(36)}{x^2}$$

$$3 = \frac{3(36)}{x^2}$$

$$x^2 = 36$$

$x = 6 \qquad y = 6 \qquad \boxed{9'' \times 9''}$

7.7 Indeterminate Forms & L'Hôpital's Rule

$\boxed{\frac{0}{0}, \ \frac{\infty}{\infty},}$ $0 \cdot \infty, \ 1^{\infty}, \ 0^0, \ and \ \infty - \infty$ are called indeterminate forms.

L'Hôpital's Rule:

Let f and g be functions that are differentiable on an open interval (a, b) containing c, except possibly at c itself. Assume that $g'(x) \neq 0$ for all x in (a, b), except possibly at c itself. If the limit of f(x)/g(x) as x approaches c produces an indeterminate form 0/0, ∞/∞, (−∞)/∞, or ∞/(−∞), then

$$\lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)}$$

7.7

12. $\lim\limits_{x\to -1}\dfrac{x^2-x-2}{x+1} = \lim\limits_{x\to -1}\dfrac{(x-2)(x+1)}{x+1}$

$= \lim\limits_{x\to -1}(x-2) = -1-2 = \boxed{-3}$

$= \lim\limits_{x\to -1}\dfrac{2x-1}{1}$

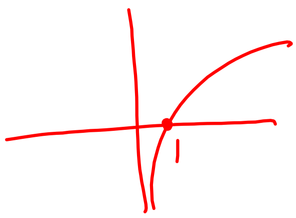
$= 2(-1)-1 = \boxed{-3}$

16. $$\lim_{x \to 0^+} \frac{e^x - (1+x)}{x^3} = \frac{0}{0} \text{ L'H applies}$$

$$= \lim_{x \to 0^+} \frac{e^x - 1}{3x^2} = \frac{0}{0} \text{ L'H}$$

$$= \lim_{x \to 0^+} \frac{e^x \to 1}{6x \to 0} = \boxed{\infty}$$

18. $$\lim_{x \to 1} \frac{\ln x^2}{x^2 - 1}$$



$$= \lim_{x \to 1} \frac{\frac{1}{x^2} \cdot \cancel{2x}}{\cancel{2x}}$$

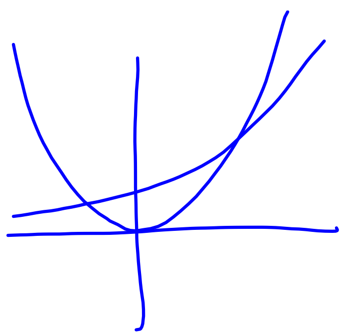
$$= \frac{1}{1} = \boxed{1}$$

20. $\displaystyle\lim_{x\to 0} \frac{\sin ax}{\sin bx}$

$$= \lim_{x\to 0} \frac{a\cos ax}{b\cos bx}$$

$$= \frac{a\cdot 1}{b\cdot 1} = \boxed{\frac{a}{b}}$$

28. $\displaystyle\lim_{x\to\infty} \frac{x^2}{e^x} = \frac{\infty}{\infty}$



$$= \lim_{x\to\infty} \frac{2x}{e^x}$$

$$= \lim_{x\to\infty} \frac{2}{e^x} = \boxed{0}$$

HW #9 (due Fri, 23 Jan)
3.1 # 17-31 odd - Absolute Extrema on an Interval
3.2 # 7-19 odd - Rolle's Theorem
3.2 # 31-37 odd - Mean Value Theorem
Quiz #6 - Mon, 26 Jan

HW #10 (due Fri, 30 Jan)
3.3 # 11-31 odd - Increasing, Decreasing, and Relative Extrema
3.4 #11-25 odd - Inflection Points and Concavity
3.5 #15-31 odd - Limits at Infinity
Quiz #7 - Fri, 30 Jan

HW #11 (due Wed, 4 Feb)
3.7 #3,5,17,23,29 - Optimization
7.7 #11-35 odd - l'Hopital's Rule
7.7 #37-53 odd - l'Hopital's Rule with logs
Test 4 - Wed, 4 Feb

Final Exam - Thurs, 12 Feb 1:00-3:00pm