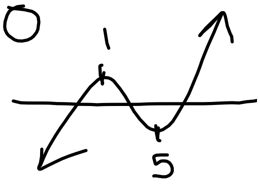
HW #9 (due Fri, 23 Jan)
3.1 # 17-31 odd - Absolute Extrema on an Interval
3.2 # 7-19 odd - Rolle's Theorem
3.2 # 31-37 odd - Mean Value Theorem
Quiz #6 - Mon, 26 Jan

HW #10 (due Fri, 30 Jan)
3.3 # 11-31 odd - Increasing, Decreasing, and Relative Extrema
3.4 #11-25 odd - Inflection Points and Concavity
3.5 #15-31 odd - Limits at Infinity
Quiz #7 - Fri, 30 Jan

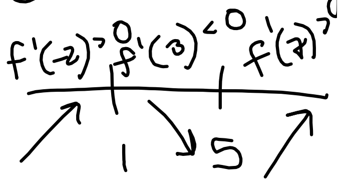
HW #11 (due Wed, 4 Feb)
3.7 #3,5,17,23,29 - Optimization
7.7 #11-35 odd - l'Hopital's Rule
7.7 #37-53 odd - l'Hopital's Rule with logs
Test 4 - Wed, 4 Feb

Final Exam - Thurs, 12 Feb 1:00-3:00pm

critical #'s : x's such that $f'(x) = 0$
or $f'(x)$ is undefined



$f(x)$ is increasing when $f'(x) > 0$
decreasing when $f'(x) < 0$



inflection pts occur when $f''(x) = 0$
f is concave up when $f''(x) > 0$
down when $f''(x) < 0$

$f^{(f)}$ continuous on $[a,b]$

& differentiable on (a,b)

Rolle's, in addition, requires

$f(a) = f(b)$

$\boxed{\frac{0}{0}, \quad \frac{\infty}{\infty}, \quad 0 \cdot \infty, \quad 1^{\infty}, \quad 0^0, \quad and \quad \infty - \infty}$ are called indeterminate forms.

L'Hôpital's Rule:

Let f and g be functions that are differentiable on an open interval (a, b) containing c, except possibly at c itself. Assume that $g'(x) \neq 0$ for all x in (a, b), except possibly at c itself. If the limit of f(x)/g(x) as x approaches c produces an indeterminate form $0/0, \infty/\infty, (-\infty)/\infty$, or $\infty/(-\infty)$, then

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

$$\lim_{x \to 0^+} x^{1/x} = \boxed{0^{\infty} \longrightarrow 0}$$

$x = 1: \quad 1^{1/1} = 1$

$x = \frac{1}{2} = \left(\frac{1}{2}\right)^{1/\frac{1}{2}} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$x = \frac{1}{10} = \left(\frac{1}{10}\right)^{1/\frac{1}{10}} = \left(\frac{1}{10}\right)^{10} = \frac{1}{10^{10}} \quad . \rightarrow \boxed{0}$

$$\lim_{x\to 0}\frac{e^x-(1-x)}{x} = \frac{0}{0}$$

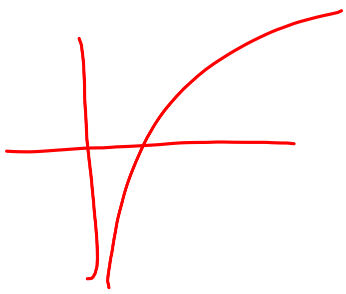
$$= \lim_{x\to 0}\frac{e^x+1}{1} = \boxed{2}$$

$$|x| = \begin{cases} x, & x\ge 0 \\ -x, & x<0 \end{cases} \qquad \sqrt[n]{x^n} = \begin{cases} |x|, & n \text{ even} \\ x, & n \text{ odd} \end{cases}$$

$$\lim_{x\to\infty}\frac{x^2}{\sqrt{x^2+1}} = \lim_{x\to\infty}\frac{x^2}{\sqrt{x^2}} = \lim_{x\to\infty}\frac{x^2}{|x|} = \lim_{x\to\infty}\frac{\cancel{x}^2}{\cancel{x}}$$

$$= \lim_{x\to\infty} x = \boxed{\infty}$$

$$\lim_{x\to\infty}\frac{\overset{[-1,1]}{\sin x}}{\underset{\infty\ \ \ \ \infty}{x-\pi}} = \boxed{0}$$



$$\lim_{x\to\infty}\frac{\ln x^4}{x^3} = \frac{\infty}{\infty}$$

$$= \lim_{x\to\infty}\frac{\frac{1}{x^{\cancel{4}\,3}}\cdot 4\cancel{x}^{\cancel{3}\,1}}{3\cancel{x}^2} = \lim_{x\to\infty}\frac{4}{3}\cdot\frac{1}{x^3} = \boxed{0}$$

$$\frac{1}{x^4}\cdot\frac{4x^3}{1}\cdot\frac{1}{3x^2}$$

$$\lim_{x\to\infty} \frac{2x^3-5x+1}{4x^3-3x^2+x+25} = \lim_{x\to\infty} \frac{2x^3}{4x^3} = \frac{2}{4} = \boxed{\frac{1}{2}}$$

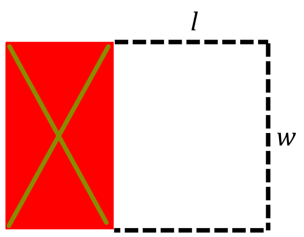
$$\lim_{x\to-\infty} \frac{-2x+5}{\sqrt{x^2+2x}} = \lim_{x\to-\infty} \frac{-2x}{|x|} = \lim_{x\to-\infty} \frac{-2x}{-x}$$

$$\sqrt{x^2}$$

$$= \boxed{2}$$

Find the horizontal asymptotes. $f(x) = \frac{5x}{\sqrt{x^2+5}}$

If I have 200 meters of fence to make a rectangular yard attached to the side of a barn, what dimensions will yield the maximum area?



$$A(\ ) = lw$$

$$200 = 2l + w$$

$$w = -2l + 200$$

$$A(l) = l(-2l + 200) = -2l^2 + 200l$$

$$A'(l) = -4l + 200$$

$$-4l + 200 = 0$$

$$200 = 4l$$

$$\boxed{50\text{m} = l}$$

$$w = -2(50) + 200$$

$$= \boxed{100\text{m}}$$

1. Locate the absolute extrema of the function on the closed interval. $f(x) = x^3 - \frac{3}{2}x^2$, $[-1, 2]$

can occur @ critical #'s & endpoints

- find $f'(x)$
- solve $f'(x) = 0$ to get critical #'s
- find $f(x)$, $f(a)$, $f(b)$ } for all critical #'s & endpoints

largest y-value is abs. max
smallest y-value is abs. min

2. Determine if Rolle's Theorem can be applied to f on the closed interval [a , b]. If Rolle's Theorem can be applied, find all values of c in the open interval (a, b) such that f'(c)=0.

$$f(x) = (x-3)(x+1)^2 \ , \qquad [-1, 3]$$

check to see if it applies!

- Is f cts. on [a, b]
- Is diff. on (a, b)
- Is f(a) = f(b)

} If all 3 are true, Rolle's applies

set $f'(x) = 0$ & solve for x
answer is x-values in (a, b)

3. Determine whether the Mean Value Theorem can be applied to f on the closed interval [a, b]. If the Mean Value Theorem can be applied, find all values of c in the open interval (a, b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$. $f(x) = x(x^2 - x - 2)$, $[-1,1]$

check Is f cts. on [a,b] & diff on (a,b)

find $f'(x)$

find $\frac{f(b)-f(a)}{b-a}$

set $f'(x) = \frac{f(b)-f(a)}{b-a}$

solve for x

answer is x's in (a,b)

5. Find the open intervals on which the function is increasing or decreasing and locate all relative extrema. $y = \frac{x^2}{x^2-9}$

solve $f'(x) = 0$ (or undefined) to get critical #'s

$f'(\ )$ | $f'(\ )$ | $f'(\ )$

$\nearrow$ x_1 $\searrow$ x_2 $\nearrow$

plug critical #'s into f

7. Find the points of inflection and discuss concavity of the graph of the function. $f(x) = \frac{x}{x^2+1}$

$f''(x) = 0$
(or undefined)