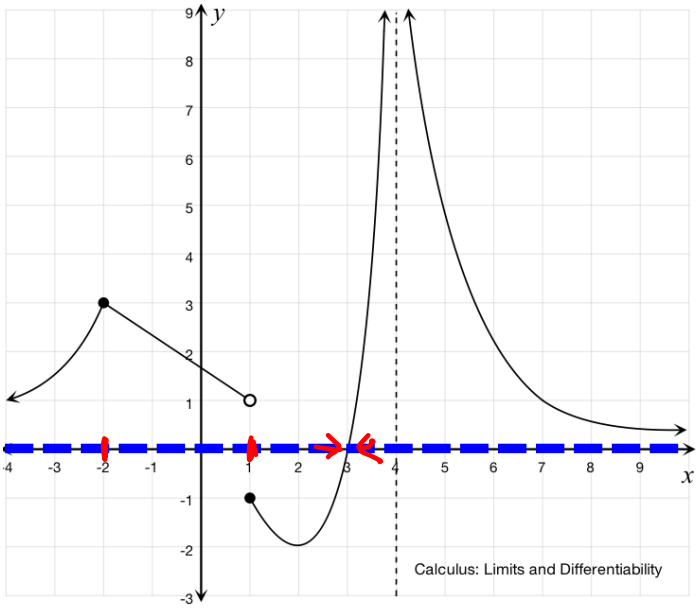
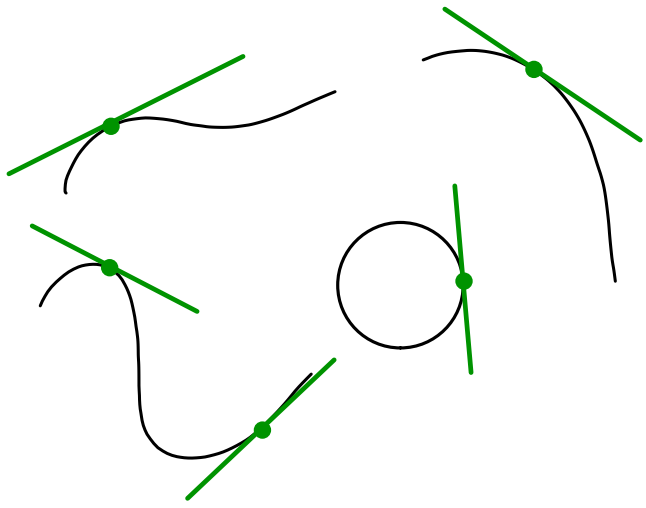
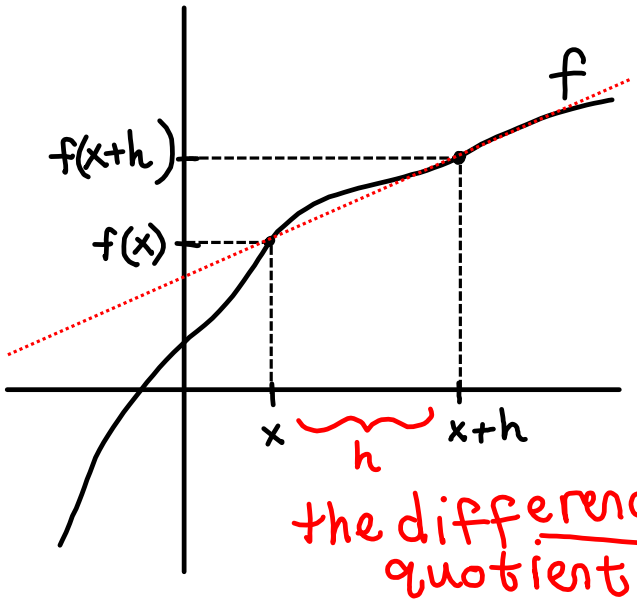


as ***x*** approaches..	***f(x)*** approaches...
-2	3
1^- (from the left)	1
1^+ (from the right)	-1
3	0
$-\infty$	0
∞	0
4	∞

tangent lines

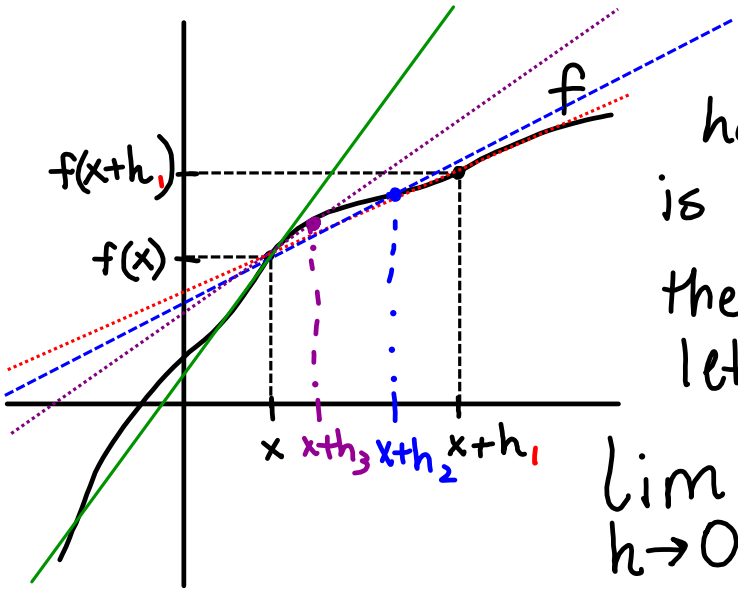




secant line

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{f(x+h) - f(x)}{x+h-x}$$

the difference quotient $= \dfrac{f(x+h)-f(x)}{h}$



tangent line

has a slope that is approximated by the difference quotient, letting h approach zero

$$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

Δx "delta x"
means change in x

$$\frac{f(x+\Delta x)-f(x)}{\Delta x} = \frac{f(x+h)-f(x)}{h}$$

Δx ↑ treated as a single variable

1.2

$$f(x)=\frac{x-2}{x^2-4}, \quad x \neq 2, -2$$

What happens to $f(x)$ as x approaches 2? $f(x) \to \frac{1}{4}$

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	0.2562	0.2506	0.2501	0.25	0.2499	0.2494	0.2439

Informal Description of the Limit

If *f(x)* becomes arbitrarily close to a single number *L* as *x* approaches *c* from either side, the limit of *f(x)*, as *x* approaches *c*, is *L*.

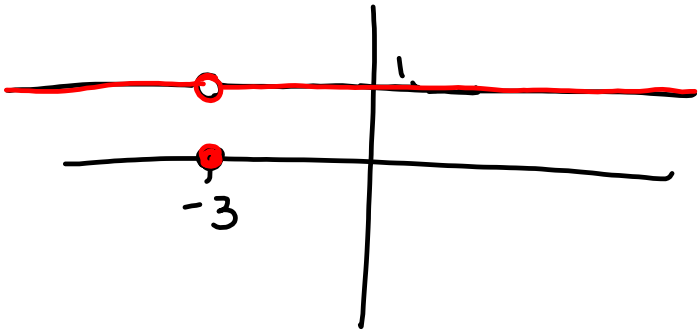
$$\lim_{x \to c} f(x) = L$$

Note: the existence or nonexistence of *f(x)* at *x=c* has no bearing on the existence of the limit as *x* approaches *c*.

A function can be undefined for a certain value of *c* with the limit as *x* approaches *c* still defined.

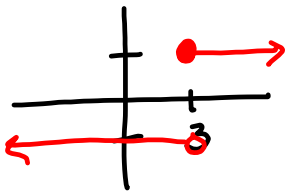
$$\lim_{x \to -3} \frac{\sqrt{1-x} - 2}{x+3} = -0.25$$

$$f(x) = \begin{cases} 1, & x \neq -3 \\ 0, & x = -3 \end{cases}$$



$$\lim_{x \to -3} f(x) = 1$$

$$\lim_{x \to 3} \frac{|x-3|}{x-3}$$



DOES NOT EXIST

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$\frac{|x-3|}{x-3} = \begin{cases} \frac{x-3}{x-3} = 1, & x-3 \geq 0, \ x \geq 3 \\ \frac{-(x-3)}{x-3} = -1, & x-3 < 0, \ x < 3 \end{cases}$$

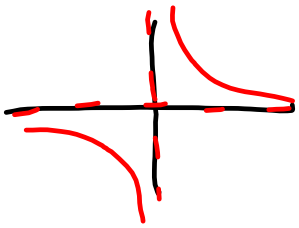
$$|x-3| = \begin{cases} x-3, & x-3 \geq 0, \ x \geq 3 \\ -(x-3), & x-3 < 0, \ x < 3 \end{cases}$$

$\lim_{x \to 3^-} f(x) = -1$; $\lim_{x \to 3^+} f(x) = 1$

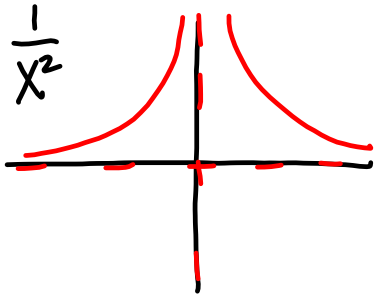
Left & right hand limits are different

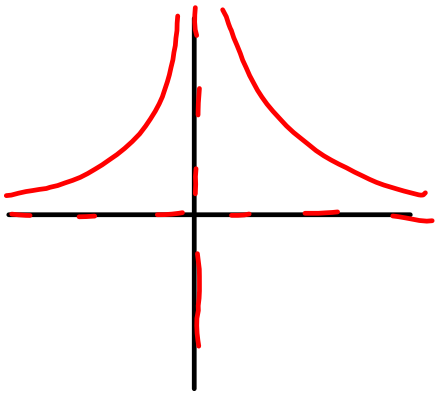
$\lim_{x \to 0} \frac{1}{x^4} = \infty$

$\frac{1}{x}$

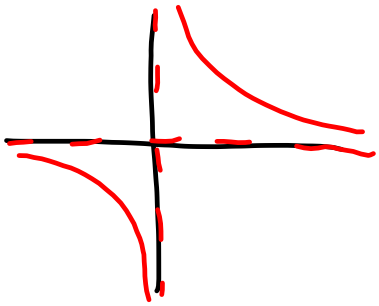


$\frac{1}{x^2}$

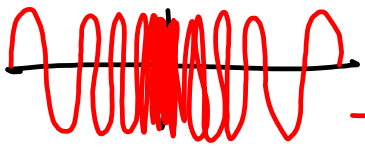


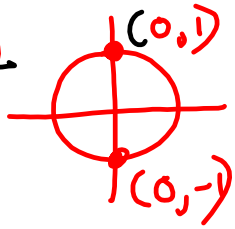


$\frac{1}{x^3}$



$\lim_{x \to 0} \sin\frac{1}{x}$ does not exist





x	$\frac{2}{\pi}$	$\frac{2}{3\pi}$	$\frac{2}{5\pi}$	$\frac{2}{7\pi}$	$\frac{2}{9\pi}$	$\frac{2}{11\pi}$
$\sin\frac{1}{x}$	$\sin\frac{\pi}{2}$	$\sin\frac{3\pi}{2}$				
	1	-1	1	-1	1	-1

"Dirichlet Function"

$$f(x) = \begin{cases} 0, & \text{if } x \text{ is rational} \\ 1, & \text{if } x \text{ is irrational} \end{cases}$$

limits do not exist anywhere!

Homework grades this week:

01: Sign up for Khan Academy with coach code 3XDPSR.

02: Read sections 1.1 and 1.2 in your textbook and complete at least 45 minutes of exercises on Khan Academy on related topics (outside of class); in addition, complete "Mastery Challenges" as often as they become available to you.

03: Textbook problems from section 1.2 #1-6 all, 15-22 all, 33,34,39,41. This will mostly be completed in class and will be due this Friday. See syllabus for proper formatting of written homework assignments.