Homework grades this week:

01: Sign up for Khan Academy with coach code 3XDPSR.

02: Read sections 1.1 and 1.2 in your textbook and complete at least 45 minutes of exercises on Khan Academy on related topics (outside of class); in addition, complete "Mastery Challenges" as often as they become available to you. page 56

03: Textbook problems from section 1.2 #1-6 all, 15-22 all, 33,34,39,41. This will mostly be completed in class and will be due this Friday. See syllabus for proper formatting of written homework assignments.

Khan Academy Exercises

Mission Foundations:

- Compare Features of Functions
- Model with Composite Functions
- Manipulate Functions

Limits Basics:

- Limits from Tables
- Limits from Graphs
- One-sided limits from graphs

Informal Description of the Limit

If *f(x)* becomes arbitrarily close to a single number *L* as *x* approaches *c* from either side, the limit of *f(x)*, as *x* approaches *c*, is *L*.

$$\lim_{x \to c} f(x) = L$$

Note: the existence or nonexistence of *f(x)* at *x=c* has no bearing on the existence of the limit as *x* approaches *c*.

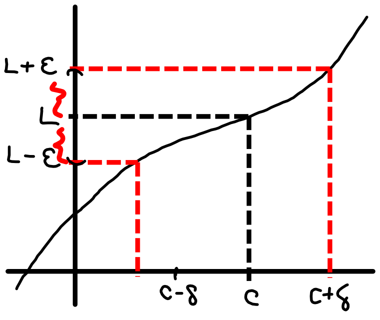
Building up to the $\epsilon - \delta$ Definition of the Limit

ε = epsilon

δ = delta

Translating the "informal description": $\lim_{x \to c} f(x) = L$

If $f(x)$ becomes arbitrarily close to a single number L as x approaches c from either side, the limit of $f(x)$, as x approaches c, is L.



"$f(x)$ becomes arbitrarily close to L"

$f(x)$ lies in the interval $(L - \varepsilon, L + \varepsilon)$ for some (really small) $\varepsilon > 0$.

$|f(x) - L| < \varepsilon$

"the distance between $f(x)$ and L is less than ε"

"x approaches c"

There exists a (very small) positive number δ such that x is either in the interval $(c - \delta, c)$ or $(c, c + \delta)$.

$0 < |x - c| < \delta$

The first inequality guarantees that $x \neq c$.

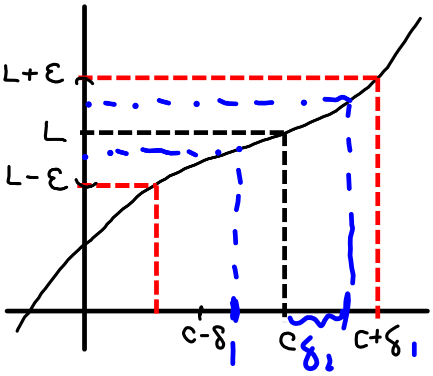
$\varepsilon - \delta$ Definition of the Limit:

Let f be a function defined on an open interval containing c (except possibly at c) and let L be a real number. The statement

$$\lim_{x \to c} f(x) = L$$

means that for each $\varepsilon > 0$, there exists a $\delta > 0$ such that if $0 < |x - c| < \delta$, then $|f(x) - L| < \varepsilon$.

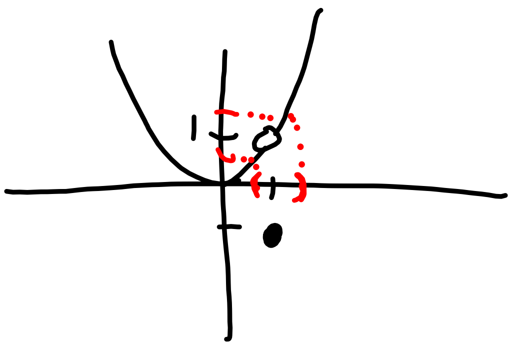
$$|x-c|<\delta \longrightarrow |f(x)-L|<\varepsilon$$



$\varepsilon - \delta$ Definition of the Limit:

$\lim_{x \to c} f(x) = L$ if given $\varepsilon > 0$, there exists a $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.

$$f(x) = \begin{cases} x^2, & x \neq 1 \\ -1, & x = 1 \end{cases}$$



$$\lim_{x \to 1} f(x) = 1$$

$\varepsilon - \delta$ Definition of the Limit:

$\lim_{x\to c} f(x) = L$ if given $\varepsilon > 0$, there exists a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.

$f(x) = 2x - 1$

Find $\lim_{x\to 4} f(x)$ and prove that is the limit using the ε-δ definition.

$L = 7$; $c = 4$; $f(x) = 2x - 1$

Let $\varepsilon > 0$ be given.

We want to find $\delta > 0$ such that $|f(x) - 7| < \varepsilon$ whenever $|x - 4| < \delta$.

$|f(x) - 7| = |2x - 1 - 7| = |2x - 8| = 2|x - 4|$

We want $2|x - 4| < \varepsilon$

$|x - 4| < \varepsilon/2$

Take $\delta = \varepsilon/2$

Then whenever $|x - 4| < \delta$, we have that

$|f(x) - 7| = 2|x - 4| < 2\delta = 2 \cdot \frac{\varepsilon}{2} = \varepsilon$

i.e. $|f(x) - 7| < \varepsilon$. Hence $\lim_{x\to 4} f(x) = 7$.

Find δ for $\varepsilon = 0.01$

24. $\lim_{x\to 4}\left(4 - \frac{x}{2}\right)$ $f(x) = 4 - \frac{x}{2}$

$c = 4$

$L = 4 - \frac{4}{2} = 2$

We want $\delta > 0$ such that $|x - c| < \delta$ guarantees us that $|f(x) - L| < 0.01$

$|f(x) - L| = \left|4 - \frac{x}{2} - 2\right| = \left|-\frac{1}{2}x + 2\right| = \left|-\frac{1}{2}(x - 4)\right|$

$= \frac{1}{2}|x - 4| < 0.01$

$|x - 4| < 0.01(2) = \boxed{0.02 = \delta}$

If $|x - 4| < 0.02$,

$|f(x) - 2| = \frac{1}{2}|x - 4| < \frac{1}{2}(0.02) = 0.01 = \varepsilon$