

This week's Assignments:

01: Sign up for Khan Academy with coach code 3XDPSR.

02: Read sections 1.1 and 1.2 in your textbook and complete at least 45 minutes of exercises on Khan Academy on related topics (outside of class); in addition, complete "Mastery Challenges" as often as they become available to you.

03: Textbook problems from section 1.2 #1-6 all, 15-22 all, 33,34,39,41. This will mostly be completed in class and will be due this Friday. See syllabus for proper formatting of written homework assignments.

Next week's Assignments:

01: **Read** sections 1.3 and 1.4 in your textbook by Monday, 22 Aug.

02: Complete at least 45 minutes of exercises on **Khan Academy** on topics related to sections 1.3 and 1.4 by Friday, 26 Aug. In addition, complete "Mastery Challenges" as often as they become available to you.

03: **Textbook exercises**, mostly be completed in class and will be due Friday, 26 Aug.

- 1.3 #11,21,27-39 odd, 41-61 odd; 83, 87 evaluating limits analytically
- 1.3 #63-73 odd; 89, 90 limits with trig, squeeze theorem
- 1.4 #1-19 odd; limits of functions with discontinuities
- 1.4 #27-30 all; 43-48 all; discuss (dis)continuity
- 1.4 #21,23,25,57,61,65,69,99,102 misc. continuity problems

1.3 Evaluating Limits Analytically

$$\text{If } \lim_{x \rightarrow c} f(x) = f(c),$$

we say that $f(x)$ is
continuous at c .



Evaluating Limits Analytically

Basic Limits

Let $b, c \in \mathbb{R}$, $n > 0$ an integer, f, g - functions, $\lim_{x \rightarrow c} f(x) = L$, $\lim_{x \rightarrow c} g(x) = K$

- | | | |
|----------------------|--|---|
| 1. Constant | $\lim_{x \rightarrow c} b = b$ | |
| 2. Identity | $\lim_{x \rightarrow c} x = c$ | $\lim_{x \rightarrow c} [f(x) \pm g(x)] = \lim_{x \rightarrow c} f(x) \pm \lim_{x \rightarrow c} g(x)$ |
| 3. Polynomial | $\lim_{x \rightarrow c} x^n = c^n$ | |
| 4. Scalar Multiple | $\lim_{x \rightarrow c} [bf(x)] = bL$ | $\lim_{x \rightarrow c} [f(x)g(x)] = \left[\lim_{x \rightarrow c} f(x) \right] \cdot \left[\lim_{x \rightarrow c} g(x) \right]$ |
| 5. Sum or Difference | $\lim_{x \rightarrow c} [f(x) \pm g(x)] = L \pm K$ | |
| 6. Product | $\lim_{x \rightarrow c} [f(x)g(x)] = LK$ | $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow c} f(x)}{\lim_{x \rightarrow c} g(x)}$, $\lim_{x \rightarrow c} g(x) \neq 0$ |
| 7. Quotient | $\lim_{x \rightarrow c} \left[\frac{f(x)}{g(x)} \right] = \frac{L}{K}$, $K \neq 0$ | |
| 8. Power | $\lim_{x \rightarrow c} [f(x)]^n = L^n$ | $\lim_{x \rightarrow c} [f(x)]^n = \left[\lim_{x \rightarrow c} f(x) \right]^n$ |

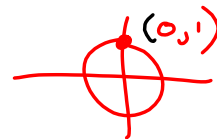
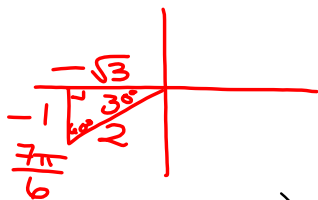
Note: If substitution yields $\frac{0}{0}$, an indeterminate form, the expression must be rewritten in order to evaluate the limit.

$\lim_{x \rightarrow c} a = a$	$\lim_{x \rightarrow 5} (-3) = -3$
$\lim_{x \rightarrow c} x = c$	$\lim_{x \rightarrow -\pi} x = -\pi$
$\lim_{x \rightarrow c} x^n = c^n$	$\lim_{x \rightarrow -1} x^5 = (-1)^5 = -1$

$$\frac{1.3}{12.} \lim_{x \rightarrow 1} (3x^3 - 2x^2 + 4) = 3(1)^3 - 2(1)^2 + 4 = 5$$

$$18. \lim_{x \rightarrow 3} \frac{\sqrt{x+1}}{x-4} = \frac{\sqrt{3+1}}{3-4} = \frac{\sqrt{4}}{-1} = -2$$

$$30. \lim_{x \rightarrow 1} \sin \frac{\pi x}{2} = \sin \frac{\pi}{2} = 1$$



$$36. \lim_{x \rightarrow 7} \sec \left(\frac{\pi x}{6} \right) = \sec \frac{7\pi}{6} = -\frac{2}{\sqrt{3}}$$

$$38. \lim_{x \rightarrow c} f(x) = \frac{3}{2} ; \lim_{x \rightarrow c} g(x) = \frac{1}{2}$$

$$(a) \lim_{x \rightarrow c} [4f(x)] = 4 \cdot \frac{3}{2}$$

$$(b) \lim_{x \rightarrow c} [f(x) + g(x)] = \frac{3}{2} + \frac{1}{2}$$

$$(c) \lim_{x \rightarrow c} [f(x)g(x)] = \left(\frac{3}{2}\right)\left(\frac{1}{2}\right)$$

$$(d) \lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{3/2}{1/2}$$

$$\lim_{x \rightarrow 3} \frac{x^2 - 2x - 3}{x - 3} = \lim_{x \rightarrow 3} \frac{\cancel{(x-3)}(x+1)}{\cancel{x-3}}$$

$$= \lim_{x \rightarrow 3} (x+1) = \boxed{4}$$

$$\begin{aligned}
 \lim_{x \rightarrow 4} \frac{x-4}{\sqrt{x}-2} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2} &= \\
 &= \lim_{x \rightarrow 4} \frac{\cancel{x-4}(\sqrt{x}+2)}{\cancel{x-4}} \\
 &= \lim_{x \rightarrow 4} (\sqrt{x}+2) = \boxed{4}
 \end{aligned}$$

Given $f(x) = 2x^2 + 3x + 1$

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\begin{aligned}
 &= \lim_{h \rightarrow 0} \frac{2(x+h)^2 + 3(x+h) + 1 - (2x^2 + 3x + 1)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{2(x^2 + 2xh + h^2) + 3x + 3h + 1 - 2x^2 - 3x - 1}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\cancel{2x^2} + 4xh + 2h^2 + \cancel{3x} + 3h + \cancel{1} - \cancel{2x^2} - \cancel{3x} - \cancel{1}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{h(4x + 2h + 3)}{h} \\
 &= \lim_{h \rightarrow 0} (4x + 2h + 3) = \boxed{4x + 3}
 \end{aligned}$$