This week's Assignments:

01: **Read** sections 1.3 and 1.4 in your textbook by Monday, 22 Aug.

02: Complete at least 45 minutes of exercises on **Khan Academy** on topics related to sections 1.3 and 1.4 by Friday, 26 Aug. In addition, complete "Mastery Challenges" as often as they become available to you.

03: **Textbook exercises**, mostly be completed in class and will be due Friday, 26 Aug.

- 1.3 #11,21,27-39 odd, 41-61 odd; 83, 87 — evaluating limits analytically
- 1.3 #63-73 odd; 89, 90 — limits with trig, squeeze theorem
- 1.4 #1-19 odd; — limits of functions with discontinuities
- 1.4 #27-30 all; 43-48 all; — discuss (dis)continuity
- 1.4 #21,23,25,57,61,65,69,99,102 — misc. continuity problems

Expect a **quiz** soon on limits.

58. $$\lim_{x\to 0} \frac{\frac{1}{x+4} - \frac{1}{4}}{x} = \lim_{x\to 0} \frac{\frac{1}{x+4}\cdot\frac{4}{4} - \frac{1}{4}\cdot\frac{x+4}{x+4}}{\frac{x}{1}}$$

$$= \lim_{x\to 0} \frac{4-(x+4)}{4(x+4)} \cdot \frac{1}{x}$$

$$= \lim_{x\to 0} \frac{-\cancel{x}^{1}}{4\cancel{x}(x+4)} = \lim_{x\to 0} \frac{-1}{4(x+4)} = \boxed{\frac{-1}{16}}$$

66. $\lim_{x\to 2} \frac{x^5 - 32}{x - 2} = \lim_{x\to 2} \frac{\cancel{(x-2)}(x^4+2x^3+4x^2+8x+16)}{\cancel{x-2}} = \boxed{80}$

$$\begin{array}{r|rrrrrr} 2 & 1 & 0 & 0 & 0 & 0 & -32 \\ & & 2 & 4 & 8 & 16 & 32 \\ \hline & 1 & 2 & 4 & 8 & 16 & \boxed{0} \end{array}$$

$$\begin{array}{r} x^4 + 2x^3 \\ x-2 \,\overline{\big)\, x^5 - 32} \\ -(x^5 - 2x^4) \\ \hline 2x^4 - 32 \\ -(2x^4 - 4x^3) \\ \cdots \end{array}$$

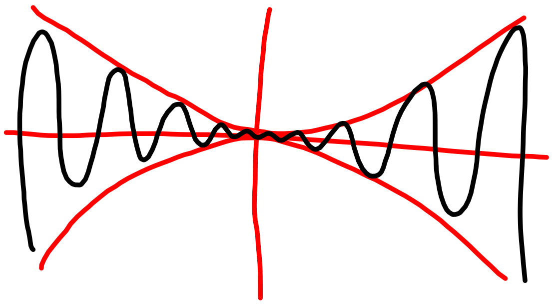
The Squeeze Theorem:

If $f(x) \leq g(x) \leq h(x)$ *and* $\lim_{x\to c} f(x) = L = \lim_{x\to c} h(x)$,

Then $\lim_{x\to c} g(x) = L$.

$-1 \leq \sin x \leq 1$

$-x^2 \leq x^2 \sin x \leq x^2$



Special Limits Derived by Squeeze Theorem:

$$\boxed{\lim_{x\to 0} \frac{\sin x}{x} = 1} \; ; \; \boxed{\lim_{x\to 0} \frac{1 - \cos x}{x} = 0}$$

Memorize!!

Use the squeeze theorem to find

$$\lim_{x\to 0}\left(x^2\cos\frac{5}{x}-3\right)$$

$$-1 \le \cos\frac{5}{x} \le 1$$

$$-x^2 \le x^2\cos\frac{5}{x} \le x^2$$

$$-x^2-3 \le x^2\cos\frac{5}{x}-3 \le x^2-3$$

$$\lim_{x\to 0}(-x^2-3) \le \lim_{x\to 0}\left(x^2\cos\frac{5}{x}-3\right) \le \lim_{x\to 0}(x^2-3)$$

$$-3 \le \lim_{x\to 0}\left(x^2\cos\frac{5}{x}-3\right) \le -3$$

Therefore, $\lim_{x\to 0}\left(x^2\cos\frac{5}{x}-3\right) = \boxed{-3}$

68. $\lim_{x\to 0}\frac{3(1-\cos x)}{x}$

$$\lim_{x\to 0}\frac{\sin x}{x} = 1$$

$$\lim_{x\to 0}\frac{1-\cos x}{x} = 0$$

$$= 3\cdot\lim_{x\to 0}\frac{1-\cos x}{x}$$

$$= 3\cdot 0$$

$$= \boxed{0}$$

72. $\lim_{x\to 0} \frac{\tan^2 x}{x}$

$\lim_{x\to 0} \frac{\sin x}{x} = 1$

$= \lim_{x\to 0} \frac{\left(\frac{\sin^2 x}{\cos^2 x}\right)}{\frac{x}{1}} = \lim_{x\to 0} \frac{\sin x \sin x}{x \cos^2 x}$

$= \left(\lim_{x\to 0} \frac{\sin x}{x}\right)\left(\lim_{x\to 0} \frac{\sin x}{\cos^2 x}\right) = 1 \cdot \frac{0}{1} = \boxed{0}$

78. $\lim_{x\to 0} \frac{\sin 2x}{\sin 3x}$

$\lim_{x\to 0} \frac{\sin x}{x} = 1$

$\lim_{2x\to 0} \frac{\sin 2x}{2x} = 1$

$= \lim_{x\to 0} \frac{\frac{\sin 2x}{2x}}{\frac{\sin 3x}{3x}} \cdot \frac{2}{3}$

1

1

$= \boxed{\frac{2}{3}}$

$$\frac{\sin 2x}{\sin 3x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \cdot \frac{\frac{2}{2}}{1} \cdot \frac{1}{\frac{3}{3}}$$

$$= \frac{\frac{\sin 2x}{2x} \cdot 2}{\frac{\sin 3x}{3x} \cdot 3}$$

$$\frac{\sin 2x}{2x} \rightarrow 1 \qquad \frac{\sin 3x}{3x} \rightarrow 1$$