This week's Assignments:

01: **Read** sections 1.3 and 1.4 in your textbook by Monday, 22 Aug.

02: Complete at least 45 minutes of exercises on **Khan Academy** on topics related to sections 1.3 and 1.4 by Friday, 26 Aug. In addition, complete "Mastery Challenges" as often as they become available to you.

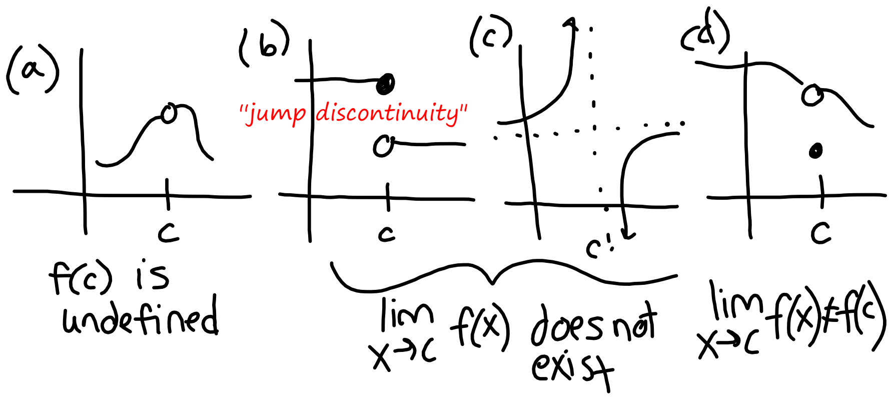
03: **Textbook exercises**, mostly be completed in class and will be due Friday, 26 Aug.

- 1.3 #11,21,27-39 odd, 41-61 odd; 83, 87 — evaluating limits analytically
- 1.3 #63-73 odd; 89, 90 — limits with trig, squeeze theorem
- 1.4 #1-19 odd; — limits of functions with discontinuities
- 1.4 #27-30 all; 43-48 all; — discuss (dis)continuity
- 1.4 #21,23,25,57,61,65,69,99,102 — misc. continuity problems

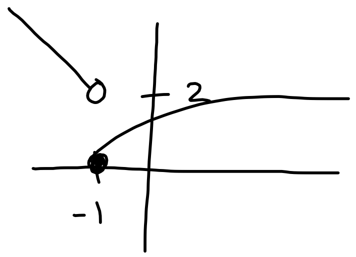
Expect a **quiz** soon on limits.

Test #1 - Next Friday??

1.4 Continuity and One-Sided Limits



These are all discontinuities
(a) and (d) are removable
(b) and (c) are nonremovable



$$\lim_{x \to -1^-} f(x) = 2$$

$$\lim_{x \to -1^+} f(x) = 0$$

$$\lim_{x \to -1} f(x) = \text{does not exist}$$

One-Sided Limits

$\lim_{x \to c^+} f(x) = L$ *limit from the right*

$\lim_{x \to c^-} f(x) = L$ *limit from the left*

$$\boxed{\begin{gathered}\lim_{x \to c} f(x) = L \text{ if and only if} \\ \lim_{x \to c^-} f(x) = L = \lim_{x \to c^+} f(x)\end{gathered}}$$

Continuity at a point

A function f is <u>continuous at c</u> if the following 3 conditions are met:

1. $f(c)$ is defined
2. Limit of $f(x)$ exists when x approaches c
3. Limit of $f(x)$ when x approaches c is equal to $f(c)$

f(x) is continuous at c if

$$\lim_{x \to c} f(x) = f(c)$$

Continuity on an open interval

A function is <u>continuous on an open interval</u> if it is continuous at each point in the interval. A function that is continuous on the entire real line $(-\infty, \infty)$ is <u>everywhere continuous</u>.

Continuity on a closed interval

A function f is <u>continuous on the closed interval</u> $[a, b]$ if it is continuous on the open interval I (a, b) and $\lim_{x \to a^+} f(x) = f(a)$ and $\lim_{x \to b^-} f(x) = f(b)$.

10. $\lim\limits_{x\to 4^-} \dfrac{\sqrt{x}-2}{x-4} \cdot \dfrac{\sqrt{x}+2}{\sqrt{x}+2}$

$= \lim\limits_{x\to 4^-} \dfrac{\cancel{x-4}}{(\cancel{x-4})(\sqrt{x}+2)}$

$= \lim\limits_{x\to 4^-} \dfrac{1}{\sqrt{x}+2} = \boxed{\dfrac{1}{4}}$

12. $\lim\limits_{x\to 2^+} \dfrac{|x-2|}{x-2} = \lim\limits_{x\to 2^+} \dfrac{x-2}{x-2} = \boxed{1}$ since $x>2$

$$|f(x)| = \begin{cases} f(x) & \text{if } f(x) \geq 0 \\ -f(x) & \text{if } f(x) < 0 \end{cases}$$

$|x-2| = x-2$ since $x>2$
$x-2>0$

1.4

Discuss the [dis]continuity of the function.

$$f(x) = \frac{(x+4)(x-2)}{(x-2)(x+1)} = \frac{x+4}{x+1}, \ x \neq 2$$

factors in denominator that cancel
$\Rightarrow$ holes $x=2$

factors in denominator that don't cancel
$\Rightarrow$ vertical asymptotes $x=-1$

f has a removable discontinuity when $x=2$ (hole) and a non-removable discontinuity when $x=-1$ (vertical asymptote).

f is continuous on

$(-\infty,-1) \cup (-1,2) \cup (2,\infty)$

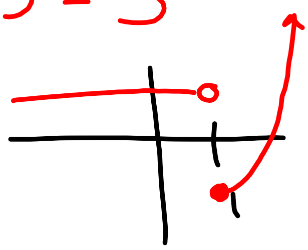
$\{x \mid x \neq -1, 2\} = \{x \mid x<-1 \text{ or } -1<x<2 \text{ or } x>2\}$

$$f(x) = \begin{cases} x^2-2, & x \geq 1 \\ 5, & x<1 \end{cases}$$

Discuss the [dis]continuity of the function.

$1^2-2 = 1-2 = -1$

$5 = 5$



f has a non-removable (jump) discontinuity @ $x=1$.

f is continuous on $(-\infty,1) \cup [1,\infty)$

$$f(x) = \begin{cases} x+6, & x \le -2 \\ x^2, & -2 < x \le 3 \\ 8 & x > 3 \end{cases}$$

Discuss the [dis]continuity of the function.

$-2+6=4$
$(-2)^2=4$

no discontinuity @ -2

$3^2=9$
$8 \neq$

f has a non-removable (jump) discontinuity @ $x=3$.
f is continuous on $(-\infty, 3] \cup (3, \infty)$