This week's Assignments:

01: **Read** sections 1.3 and 1.4 in your textbook by Monday, 22 Aug.

02: Complete at least 45 minutes of exercises on **Khan Academy** on topics related to sections 1.3 and 1.4 by Friday, 26 Aug. In addition, complete "Mastery Challenges" as often as they become available to you.

03: **Textbook exercises**, mostly be completed in class and will be due Friday, 26 Aug.

- 1.3 #11,21,27-39 odd, 41-61 odd; 83, 87 — evaluating limits analytically
- 1.3 #63-73 odd; 89, 90 — limits with trig, squeeze theorem
- 1.4 #1-19 odd; — limits of functions with discontinuities
- 1.4 #27-30 all; 43-48 all; — discuss (dis)continuity
- 1.4 #21,23,25,57,61,65,69,99,102 — misc. continuity problems

Expect a **quiz** soon on limits.

Test #1 - Next Friday??

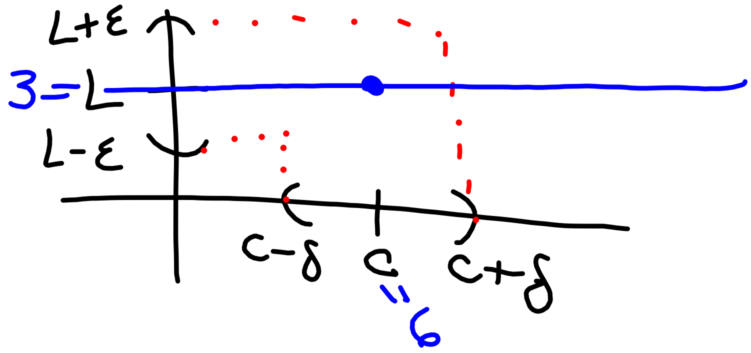
Squeeze Theorem

If $f(x) \le g(x) \le h(x)$

and $\lim_{x \to c} f(x) = \lim_{x \to c} h(x) = L$

the $\lim_{x \to c} g(x) = L$

$$\lim_{x\to 6} 3 = 3$$



Let $\delta = \varepsilon$

Then if $|x-6| < \delta$, we have that

$$|f(x) - L| = |3-3| = 0$$

$0 < \varepsilon$ always

53. $$\lim_{x\to 4} \frac{\left(\sqrt{x+5} - 3\right)}{x-4} \cdot \frac{\left(\sqrt{x+5} + 3\right)}{\sqrt{x+5} + 3}$$

$$= \lim_{x\to 4} \frac{x+5-9}{(x-4)\left(\sqrt{x+5}+3\right)}$$

$$= \lim_{x\to 4} \frac{\cancel{x-4}\ 1}{\cancel{(x-4)}\left(\sqrt{x+5}+3\right)}$$

$$= \lim_{x\to 4} \frac{1}{\sqrt{x+5}+3} = \frac{1}{\sqrt{4+5}+3} = \boxed{\frac{1}{6}}$$

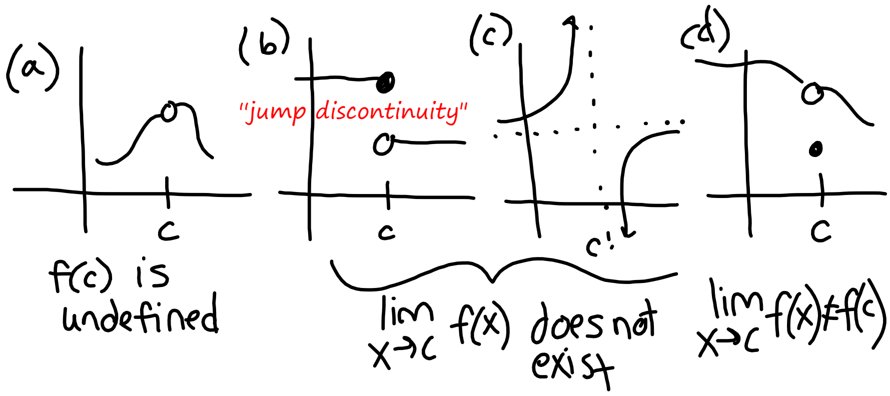
61. $$\lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - 2(x+\Delta x) + 1 - (x^2 - 2x + 1)}{\Delta x}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - 2(x+h) + 1 - (x^2 - 2x + 1)}{h}$$

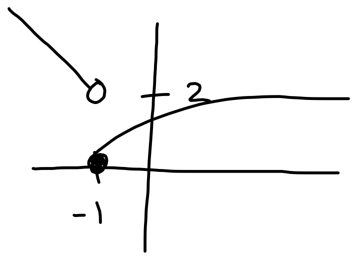
$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{2x} - 2h + \cancel{1} - \cancel{x^2} + \cancel{2x} - \cancel{1}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x + h - 2)}{\cancel{h}} = 2x - 2$$

1.4 Continuity and One-Sided Limits



These are all discontinuities
(a) and (d) are removable
(b) and (c) are nonremovable



$$\lim_{x \to -1^-} f(x) =$$

$$\lim_{x \to -1^+} f(x) =$$

$$\lim_{x \to -1} f(x) =$$

One-Sided Limits

$\lim_{x \to c^+} f(x) = L \quad \textit{limit from the right}$

$\lim_{x \to c^-} f(x) = L \quad \textit{limit from the left}$

$$\boxed{\begin{gathered}\lim_{x \to c} f(x) = L \ \textit{if and only if} \\ \lim_{x \to c^-} f(x) = L = \lim_{x \to c^+} f(x)\end{gathered}}$$

Continuity at a point

A function f is <u>continuous at c</u> if the following 3 conditions are met:

1. $f(c)$ is defined
2. Limit of $f(x)$ exists when x approaches c
3. Limit of $f(x)$ when x approaches c is equal to $f(c)$

f(x) is continuous at c if

$$\lim_{x \to c} f(x) = f(c)$$

Continuity on an open interval

A function is <u>continuous on an open interval</u> if it is continuous at each point in the interval. A function that is continuous on the entire real line $(-\infty, \infty)$is <u>everywhere continuous</u>.

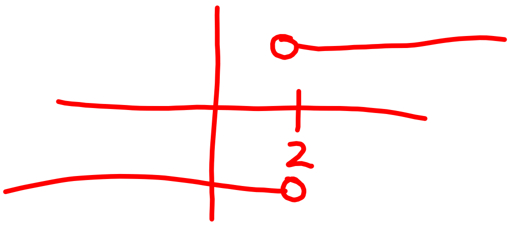
Continuity on a closed interval

A function f is <u>continuous on the closed interval</u> $[a, b]$ if it is continuous on the open interval I (a, b) and $\lim_{x \to a^+} f(x) = f(a)$ and $\lim_{x \to b^-} f(x) = f(b)$.

$$f(x) = \frac{|x-2|}{x-2}$$

Discuss the [dis]continuity of the function.

$$= \begin{cases} 1, & x>2 \\ -1, & x<2 \end{cases}$$



f is continuous on $(-\infty,2)\cup(2,\infty)$

f has a non-removable jump discontinuity @ $x=2$.

$$f(x) = \begin{cases} \frac{|x-3|}{3-x}, & |x-3|>5 \\ x^2-3, & -2\le x\le 8 \end{cases}$$

Discuss the [dis]continuity of the function.

$|x-3|>5$ $\boxed{\underset{\text{and}}{\overset{<}{\cap}}}$

$x-3>5$ $\overset{\cup}{\text{or}}$ $-(x-3)>5$

$x-3>5$ $\quad x-3<-5$

$x>8$ $\quad x<-2$

$$= \begin{cases} \frac{|x-3|}{3-x}, & x<-2 \\ x^2-3, & -2\le x\le 8 \\ \frac{|x-3|}{3-x}, & x>8 \end{cases}$$

$$\frac{|x-3|}{3-x} = \begin{cases} \frac{x-3}{3-x} = -1, & x-3>0,\ x>3 \\ \frac{-(x-3)}{3-x} = 1, & x-3<0,\ x<3 \end{cases}$$

$$= \begin{cases} 1, & x<-2 \\ x^2-3, & -2\le x\le 8 \\ -1, & x>8 \end{cases}$$

$(-2)^2-3=4-3=1$

$8^2-3\ne -1$

f has a single non-removable jump discontinuity @ $x=8$

f is continuous on $(-\infty,8]\cup(8,\infty)$