This week's assignments:

01: Read 1.4-1.5

02: 45 minutes of Khan Academy

03: Textbook exercises, due Friday, 2 Sept:

- 1.4 #1-19 odd; — limits of functions with discontinuities
- 1.4 #27-30 all; 43-48 all; — discuss (dis)continuity
- 1.4 #21,23,25,57,61,65,69,99,102 — misc. continuity problems
- 1.4 #87-98 all — Intermediate Value Theorem
- 1.5 #1,3,23,29-57 odd — Infinite Limits
- Ch 1 Review pp. 91-92 #3-83 odd — Review

Monday - Quiz/HW

Tues/Wed - 1.5/HW

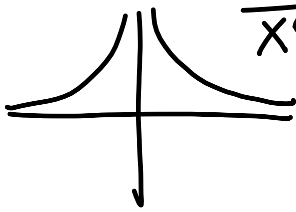
Wed/Thur - FRQ/HW

Fri - Test #1; textbook problems due

7. $$\lim_{x\to 5} \frac{\sqrt{x+4}-3}{x-5} \cdot \frac{\sqrt{x+4}+3}{\sqrt{x+4}+3}$$

$$= \lim_{x\to 5} \frac{(x+4)-9}{(x-5)(\sqrt{x+4}+3)}$$

$$= \lim_{x\to 5} \frac{\cancel{x-5}\ 1}{\cancel{(x-5)}(\sqrt{x+4}+3)} = \frac{1}{\sqrt{5+4}+3} = \frac{1}{6}$$



$\frac{1}{x^{even}}$

8. $$\lim_{x\to 0} \frac{\sin x(1-\cos x)}{2x^8}$$

$$= \lim_{x\to 0} \underbrace{\frac{\sin x}{x}}_{\downarrow\ 1} \cdot \underbrace{\frac{1-\cos x}{x}}_{\downarrow\ 0} \cdot \underbrace{\frac{1}{2x^6}}_{\downarrow\ \infty}$$

9. $$\lim_{x\to 0} \frac{\sin^4 x}{x^3}$$

$$= \lim_{x\to 0} \underbrace{\frac{\sin x}{x}}_{\searrow 1} \cdot \underbrace{\frac{\sin x}{x}}_{\searrow 1} \underbrace{\frac{\sin x}{x}}_{\searrow 1} \cdot \underbrace{\frac{\sin x}{1}}_{\searrow 0}$$

$$f(x) = \frac{x}{(x-10)(x+10)}$$

10 & -10

$$f(x) = \frac{(x-3)\cancel{(x-2)}}{(x-3)\cancel{(x-2)}}$$

V.A. (non-rem) @ $x=3$

removable disc. @ $x=2$

$$f(x) = \frac{(x-2)\cancel{(x-3)}}{(x-3)\cancel{(x-3)}}$$

non-removable disc. @ $x=3$

1.5 Infinite Limits

$$\lim_{x\to c} f(x) = \pm\infty$$

means the function increases or decreases without bound; i.e. the graph of the function approaches a vertical asymptote

Finding Vertical Asymptotes

x-values at which a function is undefined result in either holes in the graph or vertical asymptotes. Holes result when a function can be rewritten so that the factor which yields the discontinuity cancels. Factors that can't cancel yield vertical asymptotes.

Examples:

$f(x) = \frac{1}{x(x+3)}$ has vertical asymptotes at $x = 0$ and $x = 3$

$f(x) = \frac{(x+2)(x+3)}{x(x+3)}$ has a vertical asymptote at $x = 0$ and a hole at $x = -3$

Rules involving infinite limits

Let $\lim_{x\to c} f(x) = \infty$ *and* $\lim_{x\to c} g(x) = L$

1. $\lim_{x\to c}[f(x) \pm g(x)] = \infty$

2. $\lim_{x\to c}[f(x)g(x)] = \begin{cases} \infty, & L > 0 \\ -\infty, & L < 0 \end{cases}$

3. $\lim_{x\to c} \dfrac{g(x)}{f(x)} = 0$

Find the vertical asymptotes (if any).

14. $f(x) = \dfrac{-4x}{x^2+4}$

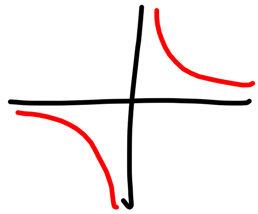
24. $h(x) = \dfrac{x^2-4}{x^3+2x^2+x+2}$

28. $g(\theta) = \dfrac{\tan\theta}{\theta}$

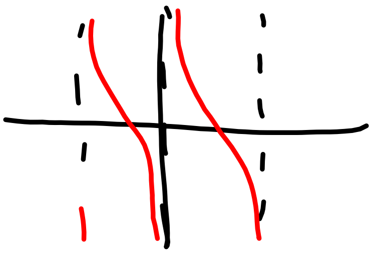
42. $\lim_{x\to0^-}\left(x^2-\frac{1}{x}\right) = \lim_{x\to0^-} x^2 - \lim_{x\to0^-}\frac{1}{x}$

$= 0-(-\infty)$

$= \boxed{\infty}$



46. $\lim_{x\to0}\frac{x+2}{\cot x} = \frac{\lim_{x\to0}(x+2)}{\lim_{x\to0}\cot x} = \frac{2}{\pm\infty} = 0$

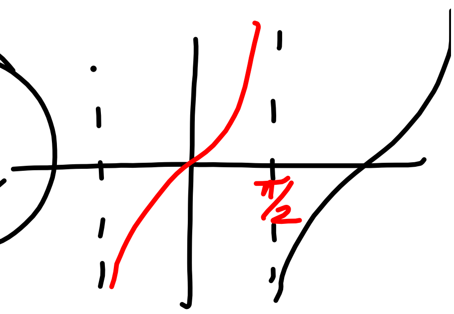


$= 2\cdot\lim_{x\to0}\tan x$

$= 2\cdot0 = 0$

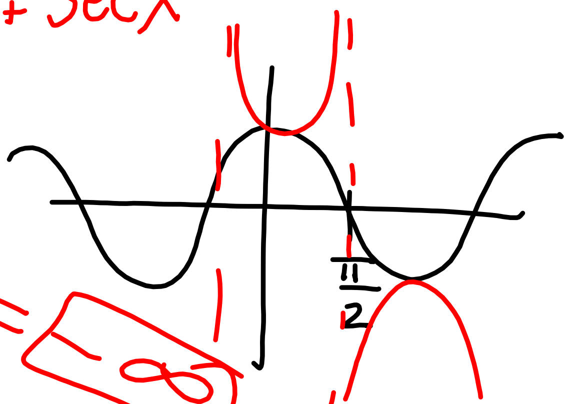
48. $\lim_{x\to\frac{1}{2}} x^2\tan\pi x = \lim_{x\to\frac{1}{2}}(x^2)\cdot\lim_{x\to\frac{1}{2}}\tan\pi x$

$= \frac{1}{4}\cdot\lim_{x\to\frac{\pi}{2}}\tan x$ does not exist



52. $\lim_{x\to3^+}\sec\frac{\pi x}{6} = \lim_{x\to\frac{\pi}{2}^+}\sec x$

$= \frac{1}{\underbrace{\lim_{x\to\frac{\pi}{2}^+}\cos x}_{\to 0}}$



$= \boxed{-\infty}$