Assignments for the Week of Sept 6:

- Read 2.1-2-2
- 45 minutes of Khan Academy
- Due Fri. 9 Sept:
  2.1 #1-41 odd; 65-89 odd

- Not due until next week:
  2.2 #3-67 odd; 87-95 odd; 97-100 all; 105,106,111,113,115

2.1 The Derivative & The Tangent Line Problem

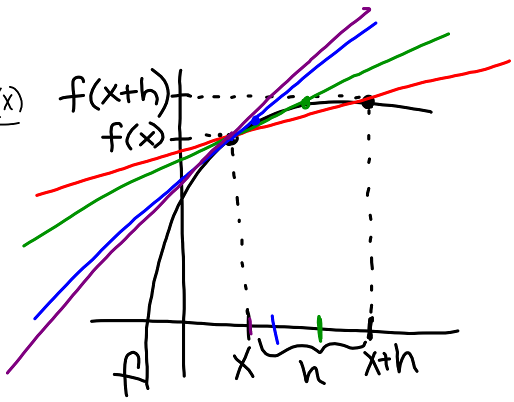
secant line crosses through a function at two points

slope of the secant line:

$$\frac{f(x+h)-f(x)}{x+h-x} = \frac{f(x+h)-f(x)}{h}$$

what happens as $h \to 0$?

$$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$



As $h \to 0$, the secant line approximates the tangent line, and the limit is the slope of the tangent line and we call it **the derivative of f at x**.

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

$\boldsymbol{f'(x)}$	"f prime of x"
$\boldsymbol{\frac{dy}{dx}}$	"derivative of y with respect to x"
$\boldsymbol{y'}$	"y prime"
$\boldsymbol{\frac{d}{dx}[f(x)]}$	"the derivative with respect to x of $f(x)$"
$\boldsymbol{D_x[y]}$	"the partial derivative with respect to x of y"

The Derivative

The slope of the tangent line to the graph of f at the point $(c, f(c))$ is given by:

$$m = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x}$$

The derivative of f at x is given by

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Find the equation of the tangent line to $f(x)=x^3-x$ at the point $(2,6)$.

$y-y_1=m(x-x_1)$

$m=f'(2)$

$$f'(x)=\lim_{h\to 0}\frac{(x+h)^3-(x+h)-(x^3-x)}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{x^3}+3x^2h+3xh^2+h^3\cancel{-x}-h\cancel{-x^3}\cancel{+x}}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{h}(3x^2+3xh+h^2-1)}{\cancel{h}}=3x^2-1$$

slope of tangent @ $(x, f(x))$

$m=3(2)^2-1=3(4)-1=11$

slope of tangent @ $(2,6)$

$y-6=11(x-2)$

$y=11x-22+6$

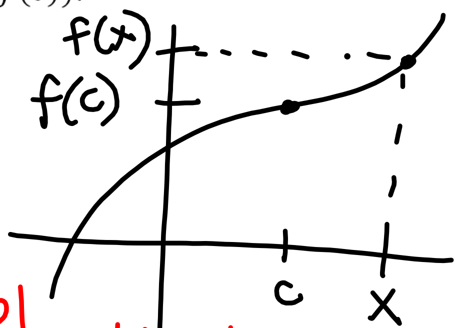
$\boxed{y=11x-16}$

equation of the tangent line to $y=x^3-x$ @ $(2,6)$

2.1 Differentiability & Continuity

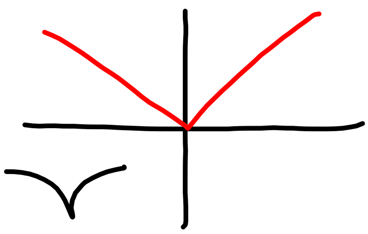
Alternative definition of the derivative at the point $(c, f(c))$:

$$f'(c)=\lim_{x\to c}\frac{f(x)-f(c)}{x-c}$$



All differentiable functions are continuous, but not all continuous functions are differentiable.

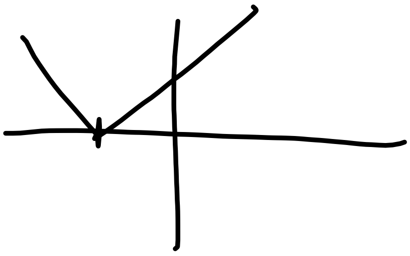
e.g. $f(x)=|x|$



$$\lim_{x\to 0^-}\frac{|x|-|0|}{x-0}=\lim_{x\to 0^-}\frac{|x|}{x}=-1$$

$$\lim_{x\to 0^+}\frac{|x|-|0|}{x-0}=\lim_{x\to 0^+}\frac{|x|}{x}=1$$

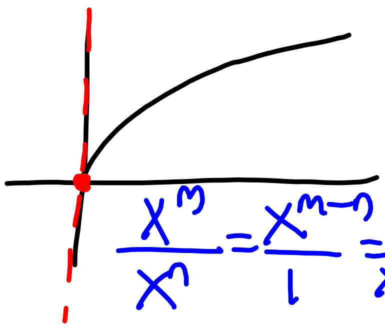
$f(x) = |x+3|$



$$\lim_{x \to -3^-} \frac{|x+3| - |-3+3|}{x-(-3)} \quad \lim_{x\to -3} \frac{|x+3|}{x+3} = -1$$

$$\lim_{x \to -3^+} = 1$$

$f(x) = \sqrt{x}$



$$\lim_{x\to 0} \frac{\sqrt{x} - \sqrt{0}}{x-0} = \lim_{x\to 0} \frac{\sqrt{x}}{x}$$

$$= \lim_{x\to 0} \frac{x^{1/2}}{x^1} = \lim_{x\to 0} \frac{1}{x^{1/2}}$$

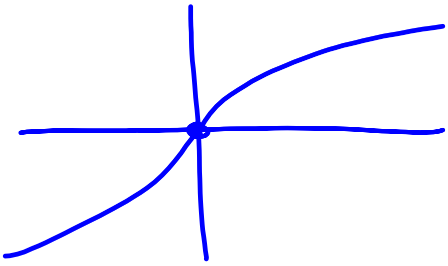
$$\frac{x^m}{x^n} = \frac{x^{m-n}}{1} = \frac{1}{x^{n-m}}$$

$$= \lim_{x\to 0} \frac{1}{\sqrt{x}} \to \infty \qquad x^{1-1/2}$$

vertical tangent line

f'(0) does not exist / undefined

$f(x) = \sqrt[3]{x}$ $\qquad f'(0) = ?$



$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

$$f'(0) = \lim_{x \to 0} \frac{\sqrt[3]{x} - \sqrt[3]{0}}{x - 0}$$

$$= \lim_{x \to 0} \frac{\sqrt[3]{x}}{x} = \lim_{x \to 0} \frac{x^{1/3}}{x^{3/3}} = \lim_{x \to 0} \frac{1}{x^{2/3}}$$

$$= \lim_{x \to 0} \frac{1}{\sqrt[3]{x^2}} = \lim_{x \to 0} \frac{1}{(\sqrt[3]{x})^2} = \infty$$

vertical tangent

$f'(0)$ undefined