Assignments for the Week of Sept 6:

- Read 2.1-2-2
- 45 minutes of Khan Academy
- Due Fri. 9 Sept:
  2.1 #1-41 odd; 65-89 odd

- Not due until next week:
  2.2 #3-67 odd; 87-95 odd; 97-100 all; 105,106,111,113,115

2.1 The Derivative & The Tangent Line Problem

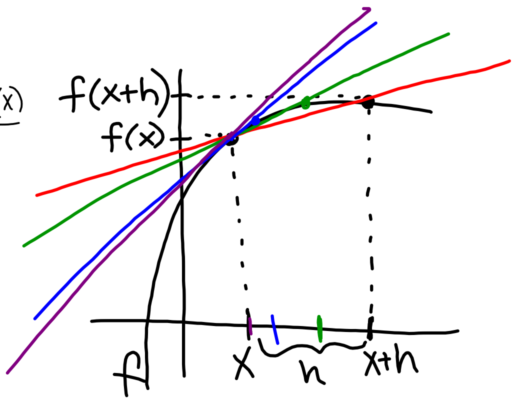
secant line
crosses through a function at two points

slope of the secant line:

$$\frac{f(x+h)-f(x)}{x+h-x} = \frac{f(x+h)-f(x)}{h}$$

what happens as $h \to 0$?

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$



As $h \to 0$, the secant line approximates the tangent line, and the limit is the slope of the tangent line and we call it **the derivative of f at x**.

The Derivative

The slope of the tangent line to the graph of f at the point $(c, f(c))$ is given by:

$$m = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x}$$

The derivative of f at x is given by

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$f'(x)$

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x}$$

symmetric difference quotient

2.1 Differentiability & Continuity

Alternative definition of the derivative at the point $(c, f(c))$:

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

All differentiable functions are continuous, but not all continuous functions are differentiable.

e.g. $f(x) = |x|$

2.2 Basic Differentiation Rules

1. The derivative of a constant function is zero, i.e.,

for $c \in \mathbb{R}$, $\frac{d}{dx}[c] = 0$

Proof:

$$\frac{d}{dx}[c] = \lim_{h\to 0} \frac{c-c}{h} = \lim_{h\to 0} \frac{0}{h} = \lim_{h\to 0} 0$$

$$= 0$$

2. Power Rule for $n \epsilon \mathbb{Q}$, $\frac{d}{dx}[x^n] = nx^{n-1}$ Special case: $\frac{d}{dx}[x] = 1$

Proof:

Recall the binomial expansion:

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2}a^{n-2}b^2 + \cdots + \frac{n!}{k!\,(n-k)!}a^{n-k}b^k + \cdots + b^n$$

$$[x^n]' = \lim_{h\to 0} \frac{(x+h)^n - x^n}{h} =$$

$$= \lim_{h\to 0} \frac{\cancel{x^n} + nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2 + \ldots + \frac{n!}{k!(n-k)!}x^{n-k}h^k + \ldots h^n - \cancel{x^n}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(nx^{n-1} + \underbrace{}_{\text{all have an } h \text{ still}})}{\cancel{h}}$$

$$= \boxed{nx^{n-1}}$$

Examples:

$[x^n]' = nx^{n-1}$ $[c]' = 0$

$\frac{d}{dx}[x^7] = 7x^6$

$\sqrt[n]{x^m} = x^{m/n}$

$\frac{d}{dx}[\pi^3] = 0$

$\frac{d}{dx}[2e] = 0$

$\frac{d}{dx}[\sqrt{x}] = \frac{d}{dx}[x^{1/2}] = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$\frac{d}{dx}\left[\frac{1}{x^3}\right] = \frac{d}{dx}[x^{-3}] = -3x^{-4} = -\frac{3}{x^4}$

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3. Constant Multiple Rule $\in \mathbb{R}$, $\frac{d}{dx}[cf(x)] = cf'(x)$

4. Sum & Difference Rules $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

Examples:

$f(x) = 3x^2$

$f'(x) = 3(x^2)' = 3(2x) = 6x$

$f(x) = \frac{3}{x} = 3x^{-1}$

$f'(x) = -3x^{-2} = -\frac{3}{x^2}$

$g(x) = 2x^3 - x^2 + 3x$

$g'(x) = 6x^2 - 2x + 3$

$y = 4x^{3/2} - 5x^4 + 2x^{\frac{1}{3}} - 7$

$y' = 6x^{1/2} - 20x^3 + \frac{2}{3}x^{-2/3}$

Derivatives of Trig Functions

1. $\frac{d}{dx}[\sin x] = \cos x$
2. $\frac{d}{dx}[\cos x] = -\sin x$
3. $\frac{d}{dx}[\tan x] = \sec^2 x$
4. $\frac{d}{dx}[\cot x] = -\csc^2 x$
5. $\frac{d}{dx}[\sec x] = \sec x \tan x$
6. $\frac{d}{dx}[\csc x] = -\csc x \cot x$

Proof that (sin x)'=cos x

$$(\sin x)' = \lim_{h\to 0} \frac{\sin(x+h)-\sin x}{h} =$$

$$= \lim_{h\to 0} \frac{\sin x\cosh + \cos x\sinh - \sin x}{h} =$$

$$= \lim_{h\to 0} \left[\frac{-\sin x+\sin x\cosh}{h} + \frac{\cos x\sinh}{h}\right]$$

$$= \lim_{h\to 0} \left[\frac{-\sin x(1-\cosh)}{h} + \cos x \cdot \frac{\sinh}{h}\right]$$

$$= (-\sin x)(0) + \cos x(1) = \boxed{\cos x}$$

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2.2

22. $y = 5 + \sin x$

$y' = 0 + \cos x = \boxed{\cos x}$

24. $y = \frac{5}{(2x)^3} + 2\cos x = \frac{5}{8}x^{-3} + 2\cos x$

$y' = \frac{-15}{8}x^{-4} - 2\sin x$