Assignments for the Week of Sept 26:

- Read 2.4, 5.1,5.4,5.5,5.6 (only derivative examples from Ch 5)
- 45 minutes of Khan Academy
- Due Fri. 30 Sept:
- 2.4 #7-33 odd; 43-89 odd — Chain rule
- 5.1 #41-59 odd; 69,71 — Logarithmic functions
- 5.4 #33-51 odd; 59, 61 — Exponential functions
- 5.5 #37-69 odd — Log and exp functions with other bases
- 5.6 #39-63 odd — Inverse trig functions

$$f(x) = \frac{1}{x^3} + \sqrt{x} - \frac{1}{\sqrt[5]{x}}$$

$$= x^{-3} + x^{1/2} - x^{-1/5}$$

$$f'(x) = \boxed{-3x^{-4} + \frac{1}{2}x^{-1/2} + \frac{1}{5}x^{-6/5}}$$

$$= \frac{-3}{x^4} + \frac{1}{2\sqrt{x}} + \frac{1}{5\sqrt[5]{x^6}}$$

$$= \frac{-3}{x^4} + \frac{1}{2\sqrt{x}} + \frac{1}{5x\sqrt[5]{x}}$$

Find $y', y'', y''', y^{(4)}, y^{(5)}, \ldots, y^{(n)}$

$y = 5x^3 - 3x^2 + 2$

$y' = 15x^2 - 6x$

$y'' = 30x - 6$

$y''' = 30$

$y^{(4)} = 0$

$y^{(5)} = 0$

. . . .

$y = x^6 + 2x^5 - 3x^4 + 2x - 5$

$y' = 6x^5 + 10x^4 - 12x^3 + 2$

$y'' = 30x^4 + 40x^3 - 36x^2$

$y''' = 120x^3 + 120x^2 - 72x$

$y^{(4)} = 360x^2 + 240x - 72$

$y^{(5)} = 720x + 240$

$y^{(6)} = 720$

$y^{(7)} = 0$

Find $y', y'', y''', y^{(4)}, y^{(5)}, \ldots, y^{(n)}$

$y = 5x^3 - 3x^2 + 2$ $\qquad$ $y = x^6 + 2x^5 - 3x^4 + 2x - 5$

If $f(x)$ is a polynomial of degree n, then $f^{(n+1)}(x) = 0$.

If $f(x) = x^n$, then

$f^{(n)}(x) = n! = n(n-1)(n-2)\cdots 3 \cdot 2 \cdot 1$

$f(x) = 3x^9 - 15x^4 + 23x^{16} - 201x^7 - 3$ $\qquad$ $f^{(17)} = 0$

Power Rule:

$$\frac{d}{dx}[x^n] = nx^{n-1} \qquad \text{d/dx [c]=0}$$

Product Rule:

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

Constant Multiple Rule:

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$$

Quotient Rule:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$$

Sum & Difference:

$$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$$

Chain Rule:

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

Trig Functions:

$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\tan x] = \sec^2 x \qquad \frac{d}{dx}[\sec x] = \sec x \tan x$$

$$\frac{d}{dx}[\cos x] = -\sin x \qquad \frac{d}{dx}[\cot x] = -\csc^2 x \qquad \frac{d}{dx}[\csc x] = -\csc x \cot x$$

$$f(x) = \cot(5x^2-3x)$$

$$f'(x) = \left[-\csc^2(5x^2-3x)\right]\cdot(10x-3)$$

$$f(x) = \sqrt[3]{\csc(4x)}$$

$$= [\csc(4x)]^{1/3}$$

$$f'(x) = \boxed{\frac{1}{3}[\csc 4x]^{-2/3} \cdot (-\csc 4x \cot 4x) \cdot 4}$$

$$f(x) = \frac{\sin 2x}{x^3}$$

$$= (\sin 2x)(x^{-3})$$

$$f'(x) = (\sin 2x)'(x^{-3}) + (\sin 2x)(x^{-3})'$$

$$= \boxed{(2\cos 2x)(x^{-3}) + (\sin 2x)(-3x^{-4})}$$

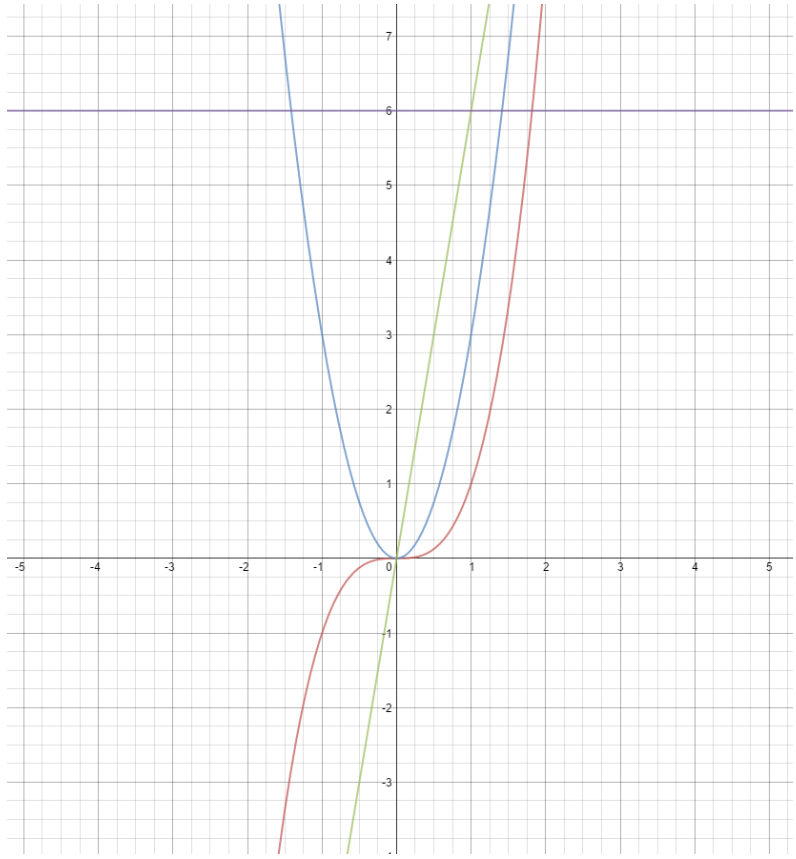
https://www.desmos.com/calculator

$y = x^3$

$y = 3x^2$

$y = 6x$

$y = 6$



Note that the derivative of a function is a function whose output at a particular value is the slope of the original function at that value.

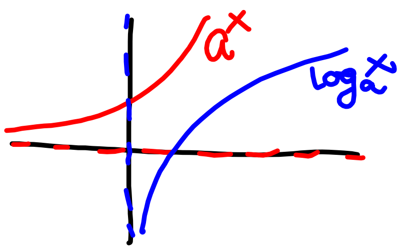
Ch 5 - Derivatives of Logarithmic and Exponential Functions

recall: $\ln x = \log_e x$

$e \approx 2.7$

$\log_2 8 = 3 \iff 2^3 = 8$

$\log_a b = c \iff a^c = b$



$y = 2^x$

$x =$ the power to which we raise 2 to get y

$=$ the # of times we multiply 2 by itself to get y

$= \log_2 y$

$$\frac{d}{dx}\left[2^x\right] = (2^x)(\ln 2)$$

$$\frac{d}{dx}\left[a^x\right] = a^x \cdot \ln a$$

$$\frac{d}{dx}\left[\log_2 x\right] = \frac{1}{x \ln 2}$$

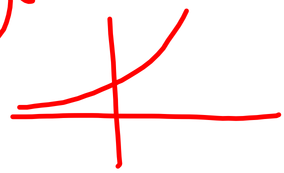
$$\frac{d}{dx}\left[\log_a x\right] = \frac{1}{x \cdot \ln a}$$

$$\frac{d}{dx}\left[\log_a u\right] = \frac{u'}{u \cdot \ln a}$$

$$\frac{d}{dx} \log_a f(x) = \frac{1}{f(x) \cdot \ln a} \cdot f'(x) = \frac{f'(x)}{f(x) \cdot \ln a}$$

$$\left[e^x\right]' = e^x \cdot \ln e = e^x \log_e e = e^x$$

$$\left[e^x\right]' = e^x$$



$$\left[\ln x\right]' = \frac{1}{x \ln e} = \frac{1}{x}$$

Since the derivative of e^x is itself, this means that graphically, at every x-value, the slope of the tangent line at that point is exactly the y-coordinate.

$$f(x) = \ln\left[\sin\left(5x^3+2x\right)\right]$$

$$f'(x) = \boxed{\frac{1}{\sin(5x^3+2x)} \cdot \cos\left(5x^3+2x\right) \cdot \left(15x^2+2\right)}$$

$$= \left(15x^2+2\right)\cot\left(5x^3+2x\right)$$

$$f(x) = (\sec x)\left(5^{\sin x}\right)$$

$$f'(x) = (\sec x)'\left(5^{\sin x}\right) + (\sec x)\left(5^{\sin x}\right)'$$

$$= \boxed{(\sec x \tan x)\left(5^{\sin x}\right) + (\sec x)\left(5^{\sin x}\ln 5\right)\cdot \cos x}$$

$$f(x) = \frac{x^2 \ln x}{\sin x}$$

$$f'(x) = \frac{(\sin x)(x^2 \ln x)' - (x^2 \ln x)(\sin x)'}{(\sin x)^2}$$

$$= \boxed{\frac{(\sin x)\left(2x \cdot \ln x + x^2 \cdot \frac{1}{x}\right) - (x^2 \ln x)(\cos x)}{\sin^2 x}}$$