Assignments for the Week of Oct 3:

- After the test,
  Read 2.5,2.6
- 45 minutes of Khan Academy by Friday
- Due Test Day:
  5.5 #37-69 odd   Log and exp functions with other bases
  5.6 #39-63 odd   Inverse trig functions
- Upcoming:
  2.5 Implicit differentiation
  2.6 related rates

2nd test: 6th per - Wed. 10/5; 8th per - Tues. 10/4

(derivatives, average & instantaneous rates of change, slope & equation of tangent lines)

Let $f(x)=\sqrt{1-\sin x}$

(a) Find $f'(x)$.

(b) Write an equation for the line tangent to the graph of f at $x=0$.

We are given the following information about two differentiable functions f and g.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	4	−3	3	2
3	6	2	−2	3

(a) $(f+g)'(3)=$

(b) $\left(\dfrac{f}{g}\right)'(1)=$

(c) If $B=f\cdot g$, then $B'(3)=$

(d) $G(x)=\sqrt{f(x)}$, then $G'(1)=$

Consider the function $f(x) = x \cdot \ln x$.

(a) Find the instantaneous rate of change of f at $x = e$. $f'(e)$

(b) Find the average rate of change of f over the interval [2.5, 3].

$$\frac{f(3) - f(2.5)}{3 - 2.5}$$

$$f'(x) = 1 \cdot \ln x + x \cdot \frac{1}{x}$$

$$f'(e) = \ln e + 1$$

$$= 1 + 1 = 2$$

The velocity of a particle moving along the x-axis is given by the equation $v(t) = \frac{\pi}{4} - \tan^{-1}(t^2 - 4t + 4)$, for $t \geq 0$. The following table gives information about $v(t)$ and $v'(t)$. $a(t) = v'(t) = \frac{-2t+4}{1+(t^2-4t+4)^2}$

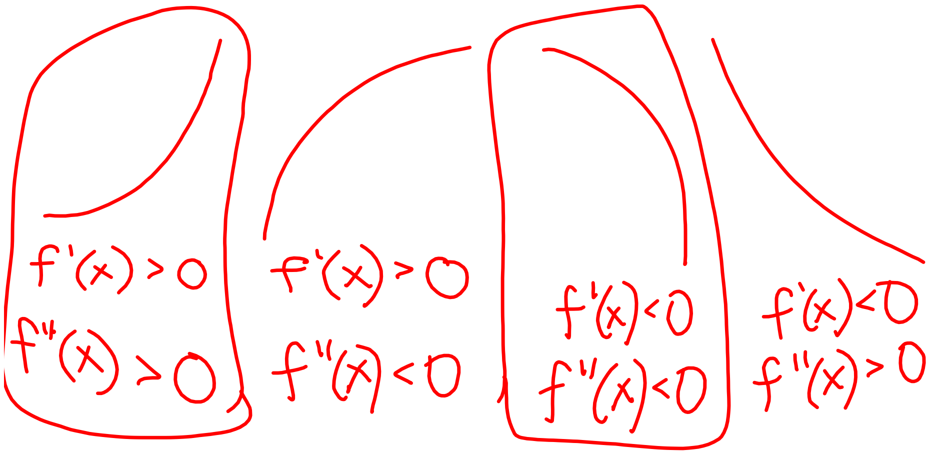
t	$0<t<1$	1	$1<t<2$	2	$2<t<3$	3	$t>3$
$v(t)$	Negative	0	Positive	Positive	Positive	0	Negative
$v'(t)$	Positive	Positive	Positive	0	Negative	Negative	Negative

(a) Find an equation for the acceleration of the particle as a function of t. You do not need to simplify the equation.

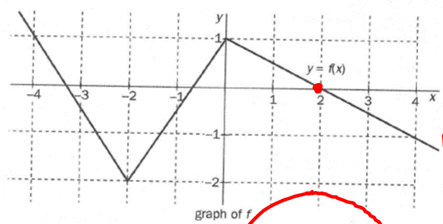
(b) Suppose that on the interval (0, 2), the particle lies in the positive ray of the x-axis. At what times in the interval (0, 2) is the particle moving away from the origin? (1, 3)

(c) During what time intervals is the speed of the particle increasing?

$1 < t < 2$, $t > 3$



The graph of a function f, shown below, consists of three line segments. Suppose $g(x)$ is a function whose derivative is f.



$f(x) = g'(x)$

$g'(-1) = f(-1) = -\frac{1}{2}$

$g'(1) = f(1) = \frac{1}{2}$

$g''(-1) = f'(-1) = \frac{3}{2}$

$g''(1) = f'(1) = -\frac{1}{2}$

(a) Find $g'(-1)$, $g'(1)$, $g''(-1)$, and $g''(1)$.

(b) Suppose $g(1) = 4$. Give an equation for the tangent line to the graph of $g(x)$ at $x = 1$.

(c) Describe the shape of the graph of $g(x)$ near $x = 2$. g has a relative maximum @ x = 2

(d) Give a piecewise defined equation for $g''(x)$.

(b) $y - 4 = -\frac{1}{2}(x - 1)$

(c) $g''(x) = \begin{cases} -3/2 , & x < -2 \\ 3/2 , & -2 < x < 0 \\ -1/2 , & x > 0 \end{cases}$

You may use a calculator for this question.

During the course of a 15-hour storm, the water levels of a reservoir are measured. In addition, some data about the rate of change of the water level is collected. The data is summarized in the table below. Assume that h and h' are both continuous and differentiable functions of t for $0 \le t \le 15$.

Time, t (hours)	2	3	8	12	15
Water level $h(t)$ (feet)	428	432	457	477	483
Rate of change $h'(t)$ (feet/hour)	**	$3.5\ \frac{ft}{h}$	$4.3\ \frac{ft}{h}$	**	$6.4\ \frac{ft}{h}$

(a) Write the equation for the tangent line to the graph of h at $t = 3$.

(b) Compute the average rate of change over the interval [2,15]. Using the Intermediate Value Theorem and the data for $h'(t)$ determine in which time intervals there must be a time where the instantaneous rate of change is equal to the average rate of change.

(c) Is the data collected about the rate of change of the water level, $h'(t)$, consistent with the statement that $h''(t) > 0$ on the interval $2 < t < 15$? Explain your answer.

(d) Is the data collected about the water level, $h(t)$, consistent with the statement that $h''(t) > 0$ on the interval $0 \le t \le 15$?Explain your answer.

$y - y_1 = m(x - x_1)$

$h(3) = 432$, $t = 3$, $h'(3) = 3.5$

$y - 432 = 3.5(x - 3)$

$\frac{h(15) - h(2)}{15 - 2} = \frac{483 - 428}{13} = 4.2$

$h'(3) < 4.2$ & $h'(8) > 4.2$ & since h' is continuous, IVT guarantees a c such that $h'(c) = 4.2$

avg ROC of h: $(2,3)$ $\frac{432 - 428}{3 - 2}$, $(3,8)$ $\frac{457 - 432}{8 - 3}$,

$(8,12)$ $\frac{477 - 457}{12 - 8}$ $(12,15)$ $\frac{483 - 477}{15 - 12}$

$$f(x) = 2^{\arcsin \pi x}$$

$$f'(x) = 2^{\arcsin \pi x} \ln 2 \cdot \frac{1}{\sqrt{1 - (\pi x)^2}} \cdot \pi$$