Assignments for the week of 10/3:

- Read 2.5-2.6
- 45 minutes of Khan Academy
- Textbook assignment due Friday, 10/7:
  2.5 # 1-39 odd; 43, 47 - Implicit Differentiation

Upcoming:
2.6 # 15-23 odd - Related Rates
2.6 # 25, 27, 35 - Related Rates (more challenging problems)
3.1 # 17-35 odd - Absolute Extrema on an Interval
3.2 # 9-21 odd - Rolle's Theorem
3.2 # 33-45 odd - Mean Value Theorem
3.3 # 17-39 odd - Increasing, Decreasing, and Relative Extrema
3.4 #15-39 odd - Inflection Points and Concavity

What happens if...

$$x^2y + y^2x = -2$$

how to find y'?

2.5 Implicit Differentiation

☆ $y = f(x)$

y is a function of x

$\frac{d}{dx}[x] = 1$; $\frac{d}{dx}[y] = y'$

6. $x^2y + y^2x = 2$

Goal: find y'

$y' = \frac{dy}{dx}$

$$\frac{d}{dx}[x^2y + y^2x] = \frac{d}{dx}[-2]$$

$$\frac{d}{dx}(x^2y) + \frac{d}{dx}(y^2x) = 0$$

$$\left[\frac{d}{dx}(x^2)\right]y + x^2\left[\frac{d}{dx}y\right] + \left[\frac{d}{dx}y^2\right]x + y^2\left[\frac{d}{dx}x\right] = 0$$

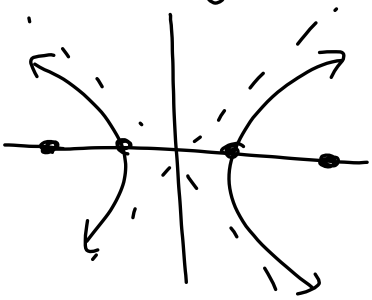
$$2xy + x^2y' + 2yy'x + y^2 \cdot 1 = 0$$

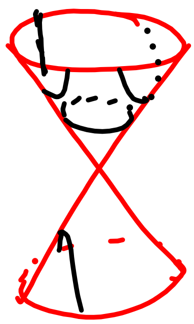
$$x^2y' + 2xyy' = -y^2 - 2xy$$

$$y'(x^2 + 2xy) = -y^2 - 2xy$$

$$\boxed{y' = \frac{-y^2 - 2xy}{x^2 + 2xy}}$$

2. $x^2 - y^2 = 16$





$$\frac{d}{dx}[x^2 - y^2] = \frac{d}{dx}[16]$$

$$2x - 2yy' = 0$$

$$-2yy' = -2x$$

$$y' = \frac{-2x}{-2y}$$

$$\boxed{y' = \frac{x}{y}}$$

8. $\sqrt{xy} = x - 2y$

$$\frac{d}{dx}\left[(xy)^{1/2}\right] = \frac{d}{dx}[x - 2y]$$

$$\frac{1}{2}(xy)^{-1/2}[1 \cdot y + x \cdot y'] = 1 - 2y'$$

$$\frac{1}{2\sqrt{xy}}[y + xy'] = 1 - 2y'$$

$$[a(b + cx) = d - ex]$$

$$\frac{y}{2\sqrt{xy}} + \frac{xy'}{2\sqrt{xy}} = 1 - 2y'$$

$$\frac{xy'}{2\sqrt{xy}} + 2y' = 1 - \frac{y}{2\sqrt{xy}}$$

$$[ax + bx = c]$$

$$y'\left[\frac{x}{2\sqrt{xy}} + 2\right] = 1 - \frac{y}{2\sqrt{xy}}$$

$$y' = \frac{1 - \frac{y}{2\sqrt{xy}}}{\frac{x}{2\sqrt{xy}} + 2}$$

10. $2\sin x\cos y = 1$

$$\frac{d}{dx}\left[(2\sin x)(\cos y)\right] = \frac{d}{dx}[1]$$

$$2\cos x\cdot\cos y + 2\sin x(-\sin y)y' = 0$$

$$2\cos x\cos y = 2y'\sin x\sin y$$

$$\frac{2\cos x\cos y}{2\sin x\sin y} = y'$$

$$\cot x\cot y = y'$$

12. $\frac{d}{dx}\left[(\sin\pi x + \cos\pi y)^2 = 2\right]$

$$2\left[\sin\pi x + \cos\pi y\right]\cdot\left(\pi\cos\pi x - \pi y'\sin\pi y\right) = 0$$

$$(2\sin\pi x + 2\cos\pi y)(\pi\cos\pi x - \pi y'\sin\pi y) = 0$$

$$2\pi\sin\pi x\cos\pi x - 2\pi y'\sin\pi x\sin\pi y + 2\pi\cos\pi x\cos\pi y - 2\pi y'\sin\pi y\cos\pi y = 0$$

$$2\pi\sin\pi x\cos\pi x + 2\pi\cos\pi x\cos\pi y = y'(2\pi\sin\pi x\sin\pi y + 2\pi\sin\pi y\cos\pi y)$$

$$y' = \frac{2\pi\sin\pi x\cos\pi x + 2\pi\cos\pi x\cos\pi y}{2\pi\sin\pi x\sin\pi y + 2\pi\sin\pi y\cos\pi y}$$

$$= \frac{\cancel{2\pi}\cos\pi x\cancel{(\sin\pi x + \cos\pi y)}}{\cancel{2\pi}\sin\pi y\cancel{(\sin\pi x + \cos\pi y)}}$$

$$y' = \frac{\cos\pi x}{\sin\pi y}$$

16. $x = \sec \frac{1}{y}$

$$\frac{d}{dx}[x] = \frac{d}{dx}\left[\sec(y^{-1})\right]$$

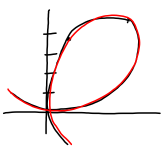
$$1 = \sec\frac{1}{y}\tan\frac{1}{y} \cdot \left(-y^{-2}y'\right)$$

$$y' = \frac{1}{-y^{-2}\sec\frac{1}{y}\tan\frac{1}{y}}$$

$$y' = -y^2\cos\frac{1}{y}\cot\frac{1}{y}$$

32. Folium of Descartes $x^3+y^3-6xy=0$

find the slope of the tangent line @ $\left(\frac{4}{3}, \frac{8}{3}\right)$



$$\frac{d}{dx}\left[x^3+y^3\right] = \frac{d}{dx}\left[6xy\right]$$

$$3x^2+3y^2y' = 6y+6xy'$$

$$3y^2y'-6xy' = 6y-3x^2$$

$$y'(3y^2-6x) = 6y-3x^2$$

$$y' = \frac{6y-3x^2}{3y^2-6x} = \frac{3(2y-x^2)}{3(y^2-2x)}$$

$$y' = \frac{2y-x^2}{y^2-2x}$$

$$m = y'\Big|_{(4/3,\,8/3)} = \frac{2\left(\frac{8}{3}\right)-\left(\frac{4}{3}\right)^2}{\left(\frac{8}{3}\right)^2-2\left(\frac{4}{3}\right)}$$

$$= \frac{\frac{16}{3}-\frac{16}{9}}{\frac{64}{9}-\frac{8}{3}} \cdot \frac{9}{9} = \frac{16(3)-16}{64-8(3)} = \frac{32}{40}$$

$$= \boxed{\frac{4}{5}}$$

40. Find y'' in terms of x & y.

$y^2 = 4x$

$2yy' = 4$

$y' = \frac{4}{2y}$

$y' = \frac{2}{y}$

$\frac{d}{dx}[y'] = \frac{d}{dx}[2y^{-1}]$

$y'' = -2y^{-2}y'$

$y'' = -2y^{-2}\left(\frac{2}{y}\right)$

$= \frac{-2}{y^2} \cdot \frac{2}{y} = \boxed{\frac{-4}{y^3}}$