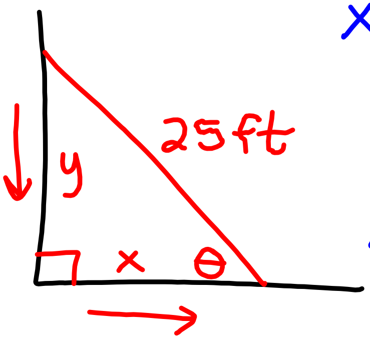
Assignments for the week of 10/3:

- Read 2.5-2.6
- 45 minutes of Khan Academy
- Textbook assignment due Friday, 10/14:
  2.5 # 1-39 odd; 43, 47 - Implicit Differentiation
  2.6 # 15-27 odd, 35 - Related Rates

- Upcoming:

3.1 # 17-35 odd - Absolute Extrema on an Interval ✓
3.2 # 9-21 odd - Rolle's Theorem ✓
3.2 # 33-45 odd - Mean Value Theorem ✓
3.3 # 17-39 odd - Increasing, Decreasing, and Relative Extrema
3.4 #15-39 odd - Inflection Points and Concavity

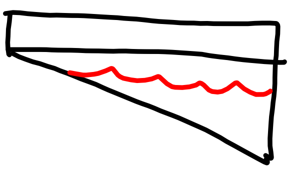
limits at infinity, l'hopital's rule, optimization

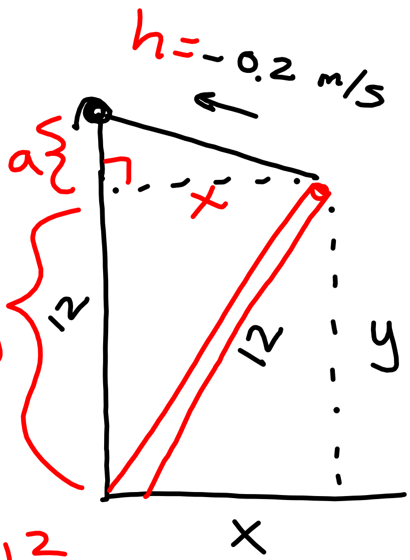


$$x^2+y^2=25^2$$

$$\tan\theta=\frac{y}{x}$$

$$\sin\theta=\frac{y}{25}$$





$$x^2+a^2=h^2$$

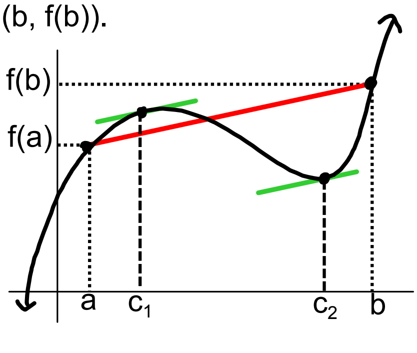
$$x^2+y^2=12^2$$

$$x^2+(12-y)^2=h^2$$

$$12^2-y^2+(12-y)^2=h^2$$

3.2 Rolle's Theorem & The Mean Value Theorem

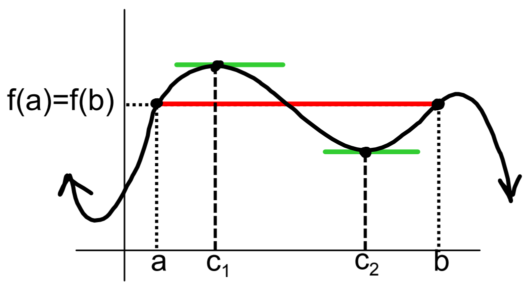
The Mean Value Thereom (MVT) states: If f is continuous on [a,b] and differentiable on (a,b), then there exists at least one c in (a,b) such that the slope of the tangent line at c is equal to the slope of the secant line through (a, f(a)) and (b, f(b)).



MVT guarantees at least one $c \in (a,b)$ such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

Rolle's Theorem is a special case of the MVT where f(a)=f(b), (and hence involving horizontal secant/tangent lines)



If $f(a) = f(b)$,

$$\frac{f(b)-f(a)}{b-a} = 0$$

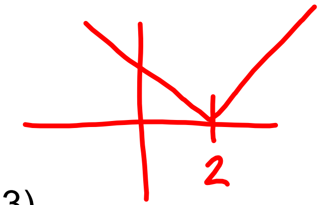
Rolle's theorem guarantees a $c \in (a,b)$ such that $f'(c) = 0$

Note that neither the Mean Value Theorem nor Rolle's Theorem apply to the following functions on the given intervals:

$$f(x) = \frac{x+5}{x-2}\ , \qquad [1,3]$$

f is not continuous on [1,3].

$$g(x) = |x-2|\ , \qquad [1,3]$$



g is continuous on [1,3], but not differentiable on (1,3).

Can Rolle's Theorem be applied?
If so, find all guaranteed values of c in (a,b).

8. $f(x) = x^2 - 5x + 4\ ,\ [1,4]$

1. Is f continuous on [1,4]? yes
2. Is f differentiable on (1,4)? yes
3. Is f(1) = f(4)? yes

$f(1) = 1^2 - 5(1) + 4 = 0$

$f(4) = 4^2 - 5(4) + 4 = 0$

Rolle's Theorem applies

[there is some $c \in (1,4)$ such that $f'(c) = 0$]

$f'(x) = 2x - 5$

$2c - 5 = 0$

$2c = 5$

$\boxed{c = 5/2}$

Can the Mean Value Theorem be applied?
If so, find all guaranteed values of c in (a,b).

34. $f(x) = \frac{x+1}{x}$, $[\frac{1}{2}, 2]$

<u>Steps to solve MVT problems</u>:
1. Is f continuous on [a,b]? yes
2. Is f differentiable on (a,b)? yes
3. Find (f(b)-f(a))/(b-a)
4. Find f'(x)
5. Set #3&4 equal, solve for x
6. Solution is the values of x from #5 that lie in (a,b)

$\}$ MVT applies $\Rightarrow$ $\exists\, c \in [\frac{1}{2}, 2]$ s.t. $f'(c) = \frac{f(2)-f(\frac{1}{2})}{2-\frac{1}{2}}$

$$\frac{f(2)-f(\frac{1}{2})}{2-\frac{1}{2}} = \frac{\frac{2+1}{2} - \frac{\frac{1}{2}+1}{\frac{1}{2}}}{2-\frac{1}{2}} = \frac{\cancel{1}+\frac{1}{2}-\cancel{1}-2}{2-\frac{1}{2}} \cdot \frac{2}{2} = \frac{1-4}{4-1} =$$

$$= -1$$

$$f(x) = \frac{x+1}{x}$$

$$f'(x) = \frac{x(1)-(x+1)(1)}{x^2} = \frac{-1}{x^2}$$

$$-1 = -\frac{1}{c^2}$$

$$c^2 = 1$$

$$c = \pm 1 \qquad -1 \notin (\tfrac{1}{2}, 2)$$

$$\boxed{c = 1}$$

38. $f(x) = 2\sin x + \sin 2x$, $[0, \pi]$

f cts on $[0,\pi]$? yes
f diff on $(0,\pi)$? yes

$\}$ MVT applies

$$\frac{f(b)-f(a)}{b-a} = \frac{(2\sin\pi + \sin 2\pi) - (2\sin 0 + \sin 2(0))}{\pi - 0}$$

$$= \frac{0+0-(0+0)}{\pi} = 0$$

$$f(x) = 2\sin x + \sin 2x$$

$$f'(x) = 2\cos x + 2\cos 2x$$

$$2\cos x + 2\underline{\cos 2x} = 0$$

$$2\cos x + 2(2\cos^2 x - 1) = 0$$

$$2\cos x + 4\cos^2 x - 2 = 0$$

$$\frac{\cancel{2}(2\cos^2 x + \cos x - 1)}{\cancel{2}} = \frac{0}{\cancel{2}}$$

$$(2\cos x - 1)(\cos x + 1) = 0$$

$$2\cos x - 1 = 0 \quad , \quad \cos x + 1 = 0$$

$$\cos x = \tfrac{1}{2} \qquad \cos x = -1$$

$$\boxed{x = \tfrac{\pi}{3}} \qquad \cancel{x = \pi} \notin (0, \pi)$$

32. $f(x) = x(x^2 - x - 2) \quad [-1, 1]$

$= x^3 - x^2 - 2x$

cts & diff on $(-\infty, \infty) \Rightarrow$ MVT applies

$$\frac{f(1) - f(-1)}{1 - (-1)} = \frac{-2 - 0}{2} = -1$$

$f'(x) = 3x^2 - 2x - 2$

$3x^2 - 2x - 2 = -1$

$3x^2 - 2x - 1 = 0$

$(3x+1)(x-1) = 0$

$x = \boxed{-1/3}, \cancel{1}$