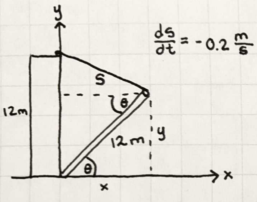
Assignments for the week of 10/3:

- Read 2.5-2.6
- 45 minutes of Khan Academy
- Textbook assignment due Friday, 10/14:
  2.5 # 1-39 odd; 43, 47 - Implicit Differentiation
  2.6 # 15-27 odd, 35 - Related Rates

- Upcoming:

3.1 # 17-35 odd - Absolute Extrema on an Interval
3.2 # 9-21 odd - Rolle's Theorem
3.2 # 33-45 odd - Mean Value Theorem
3.3 # 17-39 odd - Increasing, Decreasing, and Relative Extrema
3.4 #15-39 odd - Inflection Points and Concavity

limits at infinity, l'hopital's rule, optimization



$\frac{ds}{dt} = -0.2\,\frac{m}{s}$ Find $\frac{dx}{dt}$ & $\frac{dy}{dt}$ when $y = 6$.

Ans: $\frac{dy}{dt} = \frac{1}{5}\,\frac{m}{s}$

$\frac{dx}{dt} = \frac{-\sqrt{3}}{15}\,\frac{m}{s}$

$$\begin{cases} x^2 + y^2 = 12^2 \\ x^2 + (12-y)^2 = s^2 \end{cases}$$

$(12^2 - y^2) + (12-y)^2 = s^2$

$144 - y^2 + 144 - 24y + y^2 = s^2$

$288 - 24y = s^2$

$-24 \cdot \frac{dy}{dt} = 2s \cdot \frac{ds}{dt}$

$\frac{dy}{dt} = \frac{-1}{12} \cdot s \cdot \frac{ds}{dt}$

$\frac{dy}{dt} = \frac{-1}{12}(12)(-0.2) = \boxed{0.2\ m/s}$

when $y = 6$,

$s^2 = 288 - 24(6) = 144$

$s = 12$

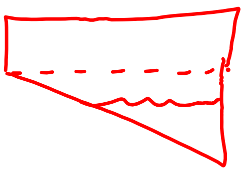
$x^2 = 144 - 6^2 = 108$

$x = 6\sqrt{3}$

$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0$

$\frac{dx}{dt} = -\frac{y}{x} \cdot \frac{dy}{dt}$

$\frac{dx}{dt} = \frac{-6}{6\sqrt{3}} \cdot 0.2 = \frac{-1}{\sqrt{3}} \cdot \frac{1}{5} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{-\sqrt{3}}{15}\,\frac{m}{s}}$



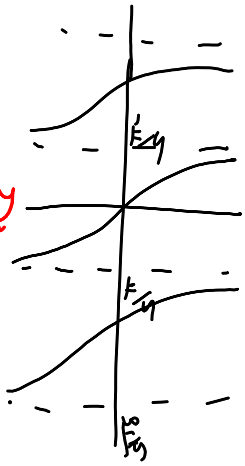
$\tan y = x$ $\tan x = y$

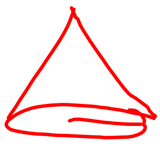
$\sec^2 y \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{\sec^2 x} = \cos^2 x$

$-\frac{\pi}{2} < y < \frac{\pi}{2}$

$x = \tan y$





$\frac{dV}{dt} = \frac{10\text{ ft}^3}{\text{min}}$; $2r = 3h$, $r = \frac{3h}{2}$

$\frac{dh}{dt} = ?$ when $h = 15$ ft

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \left(\frac{3h}{2}\right)^2 \cdot h$

$V = \frac{\pi}{3} \cdot \frac{9h^2}{4} h$

$V = \frac{3\pi}{4} h^3$

$\frac{dV}{dt} = \frac{3\pi}{4} \cdot 3h^2 \cdot \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{9\pi}{4} h^2 \cdot \frac{dh}{dt}$

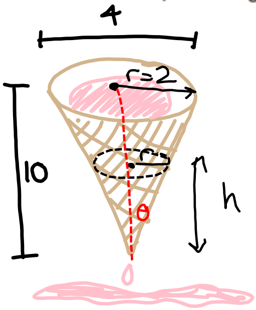
$\frac{dh}{dt} = \frac{\frac{dV}{dt}}{\frac{9\pi}{4} h^2}$

$= \frac{10}{\frac{9\pi}{4}(15)^2}$

$\frac{dh}{dt} = \frac{4 \cdot 2 \cdot \cancel{5}}{9\pi \cdot 3 \cdot 5 \cdot 3 \cdot \cancel{5}}$

$= \boxed{\frac{8}{405\pi} \text{ ft/min}}$

1. A jumbo waffle cone from Sarah's Tasty Ice Cream Shoppe is 10 inches tall and has a 4 inch diameter at the top of the cone. Yesterday, my cone had a leak! Instead of eating it super fast, I decided to compare the rate of change of volume of ice cream to the rate of change of height of ice cream in the cone. How fast is the ice cream leaking out (in cubic inches per minute) when there are 5 inches of ice cream in the cone, if the height of ice cream in the cone is changing at a rate of 1 inch every 5 minutes?



$\frac{dV}{dt} = ?\ \frac{in^3}{min}$ when $h = 5in$; $\frac{dh}{dt} = -\frac{1}{5}\frac{in}{min}$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{5}\right)^2 h$$

$$\frac{r}{h} = \frac{2}{10} \qquad r = \frac{h}{5}$$

$$V = \frac{1}{3}\pi \cdot \frac{h^2}{25} \cdot h$$

$$V = \frac{\pi}{75} h^3$$

$$\frac{d}{dt}[V] = \frac{d}{dt}\left[\frac{\pi}{75} h^3\right]$$

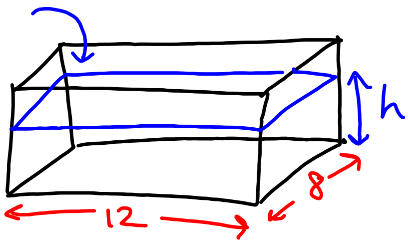
$$\frac{dV}{dt} = \frac{\pi}{75}(3h^2) \cdot \frac{dh}{dt}$$

$$\frac{dV}{dt} = \frac{\pi}{25} h^2 \frac{dh}{dt}$$

$$\frac{dV}{dt} = \frac{\pi}{25}(5in)^2 \cdot \left(\frac{-1}{5}\frac{in}{min}\right)$$

$$= \frac{\pi}{25} \cdot 25in^2 \cdot \left(\frac{-1}{5}\right)\frac{in}{min}$$

$$= \boxed{\frac{-\pi}{5}\ in^3/min}$$



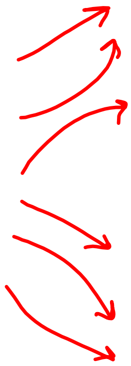
$$V = 8 \cdot 12 \cdot h$$

$$\frac{dV}{dt} = 8 \cdot 12 \cdot \frac{dh}{dt}$$

3.3-3.4 Increasing, Decreasing, Concavity, and the 1st and 2nd Derivative Tests

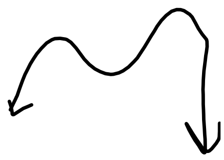
What do *f′* and *f″* tell us about *f*?

Recall that *f′* is the rate of change or slope of *f*,
f″ is the slope or rate of change of *f′*.



f'	f
+	↗ increasing
−	↘ decreasing

f''	f'	f
+	↗ increasing	concave up
−	↘ decreasing	concave down



f′(x)=0 when f has a relative maximum or minimum.
These x-values (and those where f'(x) is undefined) are called critical numbers.

f″(x)=0 when f changes concavity.
The points where concavity changes are called inflection points.