Assignments for the week of 10/17:

- Read 3.1-3.4
- 45 minutes of Khan Academy
- Textbook assignment due Friday, 10/21:
  3.1 # 17-35 odd - Absolute Extrema on an Interval
  3.2 # 11-21 odd - Rolle's Theorem
  3.2 # 33-45 odd - Mean Value Theorem
  3.3 # 23-35 odd - Increasing, Decreasing, and Relative Extrema
  3.4 #19-29 odd - Inflection Points and Concavity

upcoming:
limits at infinity, l'hopital's rule, optimization

when to have Test #3?

Friday, 10/21 $\quad$ Monday, 10/24 $\quad$ Tues(8)/Wed(6) 10/25-26

e 1. Find $\frac{dy}{dx}$ by implicit differentiation given that $2xy = 9$.

a. $\frac{dy}{dx} = -\frac{9y}{x}$

b. $\frac{dy}{dx} = -9xy$

c. $\frac{dy}{dx} = -xy$

d. $\frac{dy}{dx} = 9xy$

e. $\boxed{\frac{dy}{dx} = -\frac{y}{x}}$

$$\frac{d}{dx}[2xy] = \frac{d}{dx}[9]$$

$$(2)(y) + (2x)(y') = 0$$

$$2xy' = -2y$$

$$y' = \frac{-2y}{2x} = \frac{-y}{x}$$

B 2. For a function f, which of the following represents the instantaneous rate of change of f with respect to the variable t when $t = 2$:

A. $f(2)$

B. $\boxed{f'(2)}$

C. $f''(2)$

D. None of the above

D 3. For a function f, which of the following represents the average rate of change of f with respect to the variable t over the interval $t = 1$ to $t = 4$:

A. $\frac{f(4)+f(1)}{2}$

B. $\frac{f'(4)+f'(1)}{2}$

C. $\frac{f''(4)+f''(1)}{2}$

D. $\boxed{\frac{f(4)-f(1)}{4-1}}$

E. $\frac{f'(4)-f'(1)}{4-1}$

F. $\frac{f''(4)-f''(1)}{4-1}$

G. None of the above

D 4. A certain situation is represented by the conical volume formula $\boxed{V = \frac{1}{3}\pi r^2 h}$ where all of the variables are changing with respect to time t. If the radius is equal to half of the height, which of the following represents the rate of change of height with respect to time:

$r = \frac{h}{2}$

$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$

$= \frac{\pi}{3} \cdot \frac{h^2}{4} \cdot h$

$V = \frac{\pi}{12} h^3$

$\frac{dV}{dt} = \frac{\pi}{\cancel{12}\,4}(\cancel{3}h^2)\left(\frac{dh}{dt}\right)$

A. $\frac{dh}{dt} = \frac{\frac{dv}{dt}}{\frac{1}{2}\pi h^2}$

B. $\frac{dh}{dt} = \frac{\frac{dv}{dt}}{2\pi r^2}$

C. $\frac{dh}{dt} = \frac{\frac{dv}{dt}}{2\pi r h}$

D. $\boxed{\frac{dh}{dt} = \frac{\frac{dv}{dt}}{\frac{1}{4}\pi h^2}}$

E. $\frac{dh}{dt} = \frac{\frac{dv}{dt}}{\frac{1}{12}\pi h^2}$

F. $\frac{dh}{dt} = \frac{\frac{dv}{dt}}{2\pi r}$

G. $\frac{dh}{dt} = \frac{\frac{dv}{dt}}{\pi r^2}$

H. $\frac{dh}{dt} = \frac{\frac{dv}{dt}}{\frac{1}{6}\pi h^2}$

I. $\frac{dh}{dt} = \frac{\frac{dv}{dt}}{\frac{2}{3}\pi r h}$

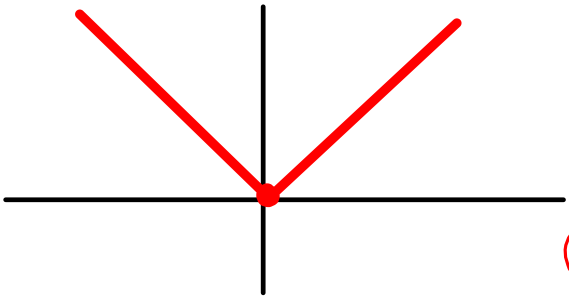
J. None of the above

D 5.

Let f be the function given by $f(x) = |x|$. Which of the following statements about f are true?

I. f is continuous at $x = 0$. ✓
II. f is differentiable at $x = 0$. ✗
III. f has an absolute minimum at $x = 0$. ✓

(A) I only (B) II only (C) III only $\boxed{\text{(D) I and III only}}$ (E) II and III only



$|x|$ is continuous @ $x=0$

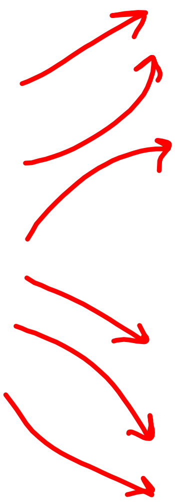
$|x|$ is not differentiable @ $x=0$

$(0,0)$ is an abs. min

3.3-3.4 Increasing, Decreasing, Concavity, and the 1st and 2nd Derivative Tests

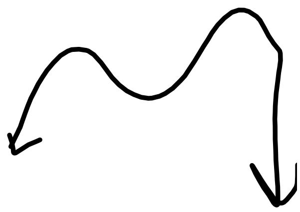
What do f' and f'' tell us about f?

Recall that f' is the rate of change or slope of f,
f'' is the slope or rate of change of f'.



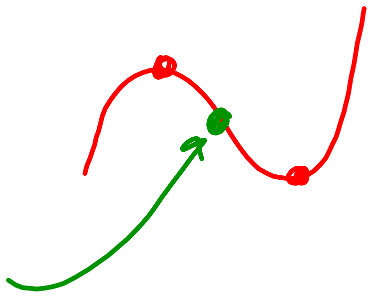
f'	f
+	↗ increasing
−	↘ decreasing

f''	f'	f
+	↗ increasing	concave up
−	↘ decreasing	concave down



f'(x)=0 when f has a relative maximum or minimum.
These x-values (and those where f'(x) is undefined) are called <u>critical numbers</u>.

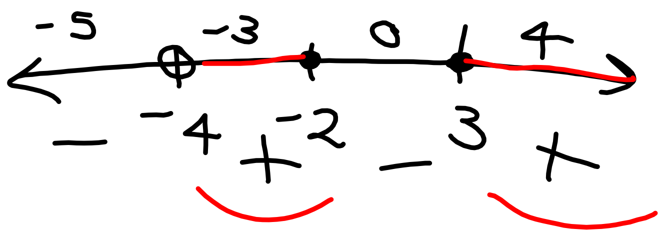
f''(x)=0 when f changes concavity.
The points where concavity changes are called <u>inflection points</u>.



To solve problems involving concavity, increasing/decreasing, etc., we should recall how to solve polynomial inequalities.

$$\frac{(x+2)(x-3)}{x+4} \geq 0$$

$$(-4,-2] \cup [3,\infty)$$



- Find all critical numbers and state the open intervals on which f is increasing and/or decreasing.
- Find all inflection points and state the open intervals on which f is concave up and/or concave down.
- Use these results to determine all relative and absolute extrema.

3.3

$f(4)=4(16-24)+15$

$f(2)=2(4-12)+15$
$=-1$

16. $f(x)=x^3-6x^2+15$

$f'(x)=3x^2-12x$
$3x(x-4)=0$
critical #'s: 0, 4
$f'(-1)$ $f'(1)$ $f'(5)$
$+$ 0 $-$ 4 $+$

f is increasing on $(-\infty,0)\cup(4,\infty)$
f is decreasing on $(0,4)$
f has a relative max @ $(0,15)$
f has a relative min @ $(4,-17)$

$f''(x)=6x-12$
$6(x-2)=0$
$f''(0)$ $f''(3)$
$-$ 2 $+$

f has an inflection point @ $(2,-1)$
f is concave down on $(-\infty,2)$
f is concave up on $(2,\infty)$

3.3

30. $f(x)=\dfrac{x+3}{x^2}$

$f'(x)=\dfrac{(x^2)(1)-(x+3)(2x)}{(x^2)^2}$
$=\dfrac{x^2-2x^2-6x}{x^4}$
$=\dfrac{-x^2-6x}{x^4}$
$=\dfrac{-x(x+6)}{x^4}$
critical #'s: $-6, 0$
$f'(-7)$ $f'(-1)$ $f'(1)$
$-$ -6 $+$ 0 $-$

f is decreasing on $(-\infty,-6)\cup(0,\infty)$
f is increasing on $(-6,0)$
f has a relative minimum @ $(-6,-\frac{1}{12})$

$f'(x)=\dfrac{-x(x+6)}{x^4}$
$=\dfrac{-x-6}{x^3}$
$f''(x)=\dfrac{x^3(-1)-(-x-6)(3x^2)}{(x^3)^2}$
$=\dfrac{-x^3+3x^3+18x^2}{x^6}$
$=\dfrac{2x^3+18x^2}{x^6}$
$=\dfrac{2x^2(x+9)}{x^6}$
$f''(x)=0$ @ $x=-9$
$f''(x)$ is undef @ $x=0$
$f''(-10)$ $f''(-1)$ $f''(1)$
$-$ -9 $+$ 0 $+$

f is concave down on $(-\infty,-9)$
f is concave up on $(-9,0)\cup(0,\infty)$
f has an inflection point @ $(-9,\frac{-2}{27})$

Leia, Han, and Luke are trapped in a rectangular room 8 feet deep and 10 feet tall. Two opposing walls are closing in at a rate of 1 foot per minute. If the water in the room is 2 feet deep when the moving walls are 12 feet apart, how fast is the water level rising when it reaches the top of Han Solo's head, if Han is 6 feet tall?