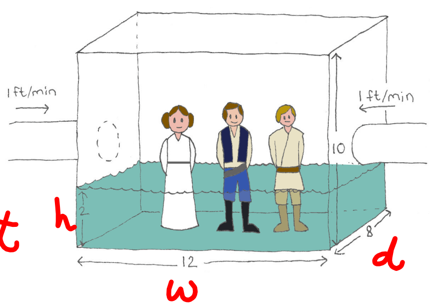
Assignments for the week of 10/17:

- Read 3.1-3.4
- 45 minutes of Khan Academy
- Textbook assignment due Friday, 10/21:
  3.1 # 17-35 odd - Absolute Extrema on an Interval
  3.2 # 11-21 odd - Rolle's Theorem
  3.2 # 33-45 odd - Mean Value Theorem
  3.3 # 23-35 odd - Increasing, Decreasing, and Relative Extrema
  3.4 #19-29 odd - Inflection Points and Concavity

upcoming:
limits at infinity, l'hopital's rule, optimization

Test #3 Monday, 10/24 - 2.5-2.6; 3.1-3.4

Leia, Han, and Luke are trapped in a rectangular room 8 feet deep and 10 feet tall. Two opposing walls are closing in at a rate of 1 foot per minute. If the water in the room is 2 feet deep when the moving walls are 12 feet apart, how fast is the water level rising when it reaches the top of Han Solo's head, if Han is 6 feet tall?



$\frac{dw}{dt} = -2\frac{ft}{min}$ $\qquad$ $\frac{dh}{dt} = ?$ when $h = 6\,ft$

$V = 2(12)(8) = 192\ ft^3$

$192 = hw \cdot 8 \rightarrow$ when $h = 6$, $w = 4$

$\boxed{24 = hw} \longrightarrow 0 = \frac{dh}{dt} w + h \cdot \frac{dw}{dt}$

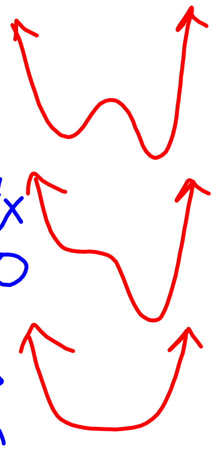
$0 = \frac{dh}{dt} \cdot 4 + 6(-2)$

$12 = \frac{dh}{dt} \cdot 4$

$\frac{dh}{dt} = \boxed{3\frac{ft}{min}}$

3.4

16. $f(x) = x^3(x-4) = x^4 - 4x^3$



$f'(x) = 4x^3 - 12x^2$

$4x^2(x-3) = 0$

critical #'s: $0, 3$

$f'(-1)$ $f'(1)$ $f'(4)$

$-$ 0 $-$ 3 $+$

f has a relative min @ $(3, -27)$

f is decreasing on $(-\infty, 3)$

f is increasing on $(3, \infty)$

$f''(x) = 12x^2 - 24x$

$12x(x-2) = 0$

$f''(-1)$, $f''(1)$, $f''(3)$

$+$ 0 $-$ 2 $+$

f is concave up on $(-\infty, 0) \cup (2, \infty)$

f is concave down on $(0, 2)$

f has inflection pts @ $(0,0)$ & $(2, -16)$

$f(x) = \dfrac{x+1}{\sqrt{x}}$ Domain: $(0, \infty)$

$$f'(x) = \frac{(\sqrt{x})(1) - (x+1)\left(\frac{1}{2\sqrt{x}}\right)}{(\sqrt{x})^2} \cdot \frac{2\sqrt{x}}{2\sqrt{x}}$$

$$= \frac{2x + x + 1}{2x^{3/2}} = \frac{3x+1}{2x^{3/2}}$$

critical #'s: $0, \cancel{-1/3}$

$\cancel{f'(-1)\ f'(-\tfrac{1}{4})}$, $f'(1)$

$-\frac{1}{3}$ 0 $+$

f is increasing on $(0, \infty)$

f is concave up on $(0, 1)$

f is concave down on $(1, \infty)$

f has an inflection point @ $(1, 2)$

$$f''(x) = \frac{(2x^{3/2})(3) - (3x+1)(3x^{1/2})}{(2x^{3/2})^2}$$

$$= \frac{6x^{3/2} - 9x^{3/2} - 3x^{1/2}}{4x^3}$$

$$= \frac{-3x^{3/2} - 3x^{1/2}}{4x^3}$$

$$= \frac{-3x^{1/2}(x-1)}{4x^3}$$

$f''(\tfrac{1}{2})$ $f''(2)$

0 $+$ 1 $-$

1. Find the <u>average</u> rate of change of the volume of a sphere <u>with respect to radius</u>, as the radius of the sphere changes from 1 cm to 2 cm.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{\Delta V}{\Delta r} = \frac{\frac{4}{3}\pi(2)^3 - \frac{4}{3}\pi(1)^3}{2-1}$$

2. Find the <u>instantaneous</u> rate of change of the volume of a sphere <u>with respect to radius</u> when the radius is 2 cm.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dr} = 4\pi r^2 \Big|_{r=2} = 4\pi(2)^2$$

3. If the radius of a sphere changes at a rate of 3cm per second, find the rate of change of the volume of the sphere with respect to time when the radius is 2 cm.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$$

$$= 4\pi(2)^2 \cdot 3$$

Find y' implicitly in terms of x and y.

$x^2y + 3xy^3 = 5x^3y^2$

$$y' = \frac{dy}{dx}$$

$$2xy + y'x^2 + 3y^3 + (3x)(3y^2y') = 15x^2y^2 + 5x^3(2yy')$$

$$y'(x^2 + 9xy^2 - 10x^3y) = 15x^2y^2 - 2xy - 3y^3$$

$$y' =$$

$\cos x + \sin y = \tan(xy)$

$-\sin x + y' \cos y = [\sec^2(xy)](1y + xy')$

$y'(\cos y - x\sec^2 xy) = y\sec^2 xy + \sin x$

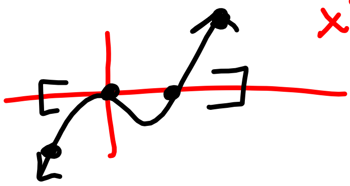
$y' = \underline{\qquad\qquad}$

1. Locate the absolute extrema of the function on the closed interval. $f(x) = x^3 - \frac{3}{2}x^2$, $[-1, 2]$

$f'(x) = 3x^2 - 3x$

$3x(x-1) = 0$

$x = 0, 1$



$x^2(x - \frac{3}{2})$

$f(-1) = -1 - 3/2 = -5/2 \leftarrow$ abs min

$f(0) = 0$

$f(1) = 1 - 3/2 = -1/2$

$f(2) = 8 - 3/2(4) = 2 \leftarrow$ abs max

2. Determine if Rolle's Theorem can be applied to f on the closed interval [a , b]. If Rolle's Theorem can be applied, find all values of c in the open interval (a, b) such that f'(c)=0.

$$f(x) = (x-3)(x+1)^2\ , \qquad [-1, 3]$$

cts on [-1,3]?
diff on (-1,3)? } yes
f(-1) = f(3)? yes } Rolle's Thm applies

find f'(x)
set f'(x) = 0
Solve for x
pick only x's in open interval

$(x-3)(x^2+2x+1)$
$x^3+2x^2+x-3x^2-6x-3$
x^3-x^2-5x-3

$f'(x) = 3x^2-2x-5$
$(3x-5)(x+1) = 0$
$x = 5/3$ $\quad \cancel{x=-1}$

3. Determine whether the Mean Value Theorem can be applied to f on the closed interval [a, b]. If the Mean Value Theorem can be applied, find all values of c in the open interval (a, b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$. $f(x) = x(x^2-x-2)\ ,\ [-1, 1]$

cts on [-1,1] ✓
diff on (-1,1) ✓ } MVT applies

find f'(x)
find $\frac{f(b)-f(a)}{b-a}$)) set them equal
& solve for x in (a,b)