Assignments for the week of 10/24:

- Read 3.5,3.7,7.7
- 45 minutes of Khan Academy
- Textbook assignment due Tuesday, 11/01 (final exam day):
  - 3.5 #15-31odd - limits at infinity
  - 3.7 #3,5,17,19,23 - optimization
  - 8.7 #11-35odd - l'Hopital's rule
  - 8.7 #47-55odd - l'Hopital's rule with logs

Test #3 Monday, 10/24 - 2.5-2.6; 3.1-3.4

FINAL EXAM - Tuesday, 11/01 - Comprehensive; can replace lowest test grade

A particle moves along the y-axis with velocity $v(t) = -\frac{2}{\pi}\sin\left(\frac{\pi}{2}t\right)$ cm/sec for time $t \geq 0$ in seconds.

(a) In what direction is the particle moving at $t = \frac{1}{3}$? What are the velocity and speed at this time? $v(1/3) < 0$ down $\left|v\left(\frac{1}{3}\right)\right|$

(b) Find the earliest time, $t_1 > 0$, when the particle changes direction. $v(t) = 0$

$t=2$

(c) What is the particle's average acceleration over the interval $[0, t_1]$? $\frac{v(2)-v(0)}{2-0}$

(d) Does the concavity of the position function, $s(t)$, change sign over the interval $[0, t_1]$?

$v'(t) = 0$

$t = 1$

5. Find the open intervals on which the function is increasing or decreasing and locate all relative extrema. $y = \frac{x^2}{x^2-9}$

$$y' = \frac{(x^2-9)(2x) - x^2(2x)}{(x^2-9)^2}$$

$$= \frac{-18x}{(x^2-9)^2}$$

critical #'s

0, 3, -3

$f'(-4)$ $f'(-1)$ $f'(1)$ $f'(4)$

$+$ -3 $+$ 0 $-$ 3 $-$

$\nearrow$ $\nearrow$ $\searrow$ $\searrow$

f is increasing on $(-\infty,-3) \cup (-3,0)$

& decreasing on $(0,3) \cup (3,\infty)$

relative max @ $(0,0)$

6. Find the points of inflection and discuss concavity of the graph of the function. $f(x) = x^3(x-4)$

$f(x) = x^4 - 4x^3$

$f'(x) = 4x^3 - 12x^2$

$f''(x) = 12x^2 - 24x$

$12x(x-2) = 0$

$f''(-1)$ $f''(1)$ $f''(3)$

$+$ 0 $-$ 2 $+$

f is concave up on

$(-\infty, 0) \cup (2, \infty)$

f is concave down on

$(0,2)$

inflection points @

$(0,0)$

$(2,-16)$

$f(x) = 2\sin x + \sin 2x \quad , \quad [0, 2\pi]$

$f'(x) = 2\cos x + 2\cos 2x$

$f''(x) = -2\sin x - 4\sin 2x$

$-2\sin x - 4\sin 2x = 0$

$-(2\sin x + 4(2\sin x\cos x)) = 0$

$2\sin x(1 + 4\cos x) = 0$

$2\sin x = 0 \qquad 1 + 4\cos x = 0$

$\sin x = 0 \qquad \cos x = -\frac{1}{4}$

$x = 0, \pi \qquad x = \_\_, \_\_$

12. The radius of a right circular cylinder is given by $\sqrt{t+2}$ and its height is $\frac{1}{2}t$, where t is time in seconds and the dimensions are in inches. Find the rate of change of the volume with respect to time. Volume of a cylinder is given by $V = \pi r^2 h$, where r is the radius of the cylinder and h is the height.

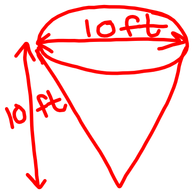
$V = \pi\left(\sqrt{t+2}\right)^2\left(\frac{1}{2}t\right)$

$= \pi(t+2)\left(\frac{1}{2}t\right)$

$V = \frac{\pi}{2}t^2 + \pi t$

$V'(t) = \boxed{\pi t + \pi}$

13. A conical tank is 10 feet across at the top and 10 feet deep. If it is being filled with water at a rate of 5 cubic feet per minute, find the rate of change of the depth of the water when it is 3 feet deep. The volume of a cone is given by $=\frac{1}{3}\pi r^2 h$, where r is the radius of the cone and h is the height. Give an exact answer in terms of π.



$\frac{dV}{dt} = 5\,\frac{ft^3}{min}$ $\frac{dh}{dt} = ?$ when $h = 3\,ft$

$\frac{2r}{h} = \frac{10}{10}$

$r = \frac{h}{2}$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$

$V = \frac{\pi h^3}{12}$

$\frac{dV}{dt} = \frac{\pi}{\cancel{12}_{4}}\left(\cancel{3}h^2\right)\cdot\frac{dh}{dt}$

$\frac{\frac{dV}{dt}}{\frac{\pi}{4}h^2} = \frac{dh}{dt}$

$\frac{4\cdot 5}{\pi\cdot 3^2} = \boxed{\frac{20}{9\pi}\ ft/min}$

3.5 Limits at Infinity

$\lim_{x\to\infty} f(x)$ (end behavior)

correspond exactly with horizontal & oblique asymptotes

$$f(x)=\frac{5x^2-3x+4}{2x^2+5x}\approx\frac{5x^2}{2x^2}=\frac{5}{2}$$

Horizontal asymptote @ $y=\frac{5}{2}$

$\lim_{x\to\infty} f(x)=\frac{5}{2}$; $\lim_{x\to-\infty} f(x)=\frac{5}{2}$

$$f(x)=\frac{2x-4}{3x^4}\approx\frac{2x}{3x^4}=\frac{2}{3x^3}\longrightarrow 0$$

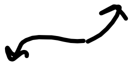
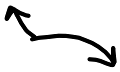
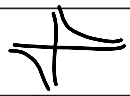
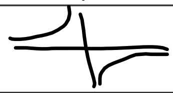
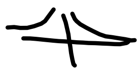
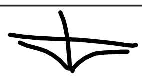
Horizontal asymptote @ $y=0$

$$f(x)=\frac{2x^7-4x^3-2}{5x^4+1}\approx\frac{2x^7}{5x^4}=\frac{2}{5}x^3$$

$\lim_{x\to\infty} f(x)=\infty$; $\lim_{x\to-\infty} f(x)=-\infty$

$$f(x)=\frac{2-7x^3+2x}{1+x}\approx\frac{-7x^3}{x}=-7x^2$$

$\lim_{x\to\infty} f(x)=-\infty$; $\lim_{x\to-\infty} f(x)=-\infty$

Ratio of lead terms of $f(x)$	Picture of graph end behavior	$\lim\limits_{x\to-\infty} f(x)$	$\lim\limits_{x\to\infty} f(x)$
$+x^{even}$		∞	∞
$-x^{even}$		$-\infty$	$-\infty$
$+x^{odd}$		$-\infty$	∞
$-x^{odd}$		∞	$-\infty$
c		c	c
$+\frac{1}{x^{odd}}$		0	0
$-\frac{1}{x^{odd}}$		0	0
$+\frac{1}{x^{even}}$		0	0
$-\frac{1}{x^{even}}$		0	0

24. $\lim\limits_{x\to-\infty}\left(\frac{1}{2}x - \frac{4}{x^2}\right)$

$= \lim\limits_{x\to-\infty} \frac{1}{2}x - \lim\limits_{x\to-\infty} \frac{4}{x^2}$

$= -\infty - 0$

$= \boxed{-\infty}$

26. $\lim\limits_{x \to -\infty} \dfrac{x}{\sqrt{x^2+1}}$

$= \lim\limits_{x \to -\infty} \dfrac{x}{\sqrt{x^2}}$

$= \lim\limits_{x \to -\infty} \dfrac{x}{|x|}$

$= \lim\limits_{x \to -\infty} \dfrac{x}{-x}$

$= \lim\limits_{x \to -\infty} (-1) = \boxed{-1}$

$$\sqrt[n]{x^n} = \begin{cases} x, & n \text{ odd} \\ |x|, & n \text{ even} \end{cases}$$

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$\lim\limits_{x \to \infty} \dfrac{5x-2}{\sqrt{9x^2+3}} = \lim\limits_{x \to \infty} \dfrac{5x}{\sqrt{9x^2}} = \lim\limits_{x \to \infty} \dfrac{5x}{3|x|}$

$= \lim\limits_{x \to \infty} \dfrac{5x}{3x} = \boxed{\dfrac{5}{3}}$

30. $\lim_{x\to\infty} \frac{x-\cos x}{x}$

bounded

$= \lim_{x\to\infty} \frac{x}{x} - \lim_{x\to\infty} \frac{\cos x}{x}$

$= 1 - \lim_{x\to\infty} \frac{[-1,1]}{x\to\infty}$

$= 1 - 0$

$= \boxed{1}$

$\lim_{x\to 0} \frac{\sin x}{x} = 1$

$\lim_{x\to\infty} \frac{\sin x}{x} = 0$

32. $\lim_{x\to\infty} \cos\frac{1}{x}$

$= \cos\left[\lim_{x\to\infty} \frac{1}{x}\right]$

$= \cos 0$

$= \boxed{1}$

18. c.

$\lim_{x\to\infty} \frac{5x^{3/2}}{4\sqrt{x}+1} = \lim_{x\to\infty} \frac{5x^{3/2}}{4x^{1/2}} =$

$= \lim_{x\to\infty} \frac{5}{4}x$

$= \infty$