Assignments for the week of 10/24:

- Read 3.5,3.7,7.7
- 45 minutes of Khan Academy
- Textbook assignment due Tuesday, 11/01 (final exam day):
  3.5 #15-31odd - limits at infinity
  3.7 #3,5,17,19,23 - optimization
  8.7 #11-35odd - l'Hopital's rule
  8.7 #47-55odd - l'Hopital's rule with logs

FINAL EXAM - Tuesday, 11/01 - Comprehensive; can replace lowest test grade

REVIEW SESSION - Monday @ 3pm?

If $x^2 + xy = 10$, then when $x = 2$, $\frac{dy}{dx} =$

$\boxed{\text{(A)} \ -\frac{7}{2}}$ (B) -2 (C) $\frac{2}{7}$ (D) $\frac{3}{2}$ (E) $\frac{7}{2}$

$$\frac{d}{dx}[x^2+xy] = \frac{d}{dx}[10]$$

$$2x + 1\cdot y + x\cdot y' = 0$$

$$xy' = -y-2x$$

$$y' = \frac{-y-2x}{x}$$

$$= \frac{-3-2(2)}{2} = \frac{-7}{2}$$

$$x^2+xy=10$$

$$2^2+2y=10$$

$$2y=6$$

$$y=3$$

The function f is given by $f(x) = x^4 + x^2 - 2$. On which of the following intervals is f increasing?

(A) $\left(-\frac{1}{\sqrt{2}}, \infty\right)$

(B) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$\boxed{\text{(C) } (0, \infty)}$

(D) $(-\infty, 0)$

(E) $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$

$f'(x) = 4x^3 + 2x$

$2x(2x^2 + 1) = 0$

$2x = 0$, $\quad 2x^2 + 1 = 0$

$\boxed{x = 0}$ critical #

$2x^2 = -1$

$x^2 = -\frac{1}{2}$

$\cancel{x = \pm\sqrt{-\frac{1}{2}}}$

$f'(-1) \quad f'(1)$

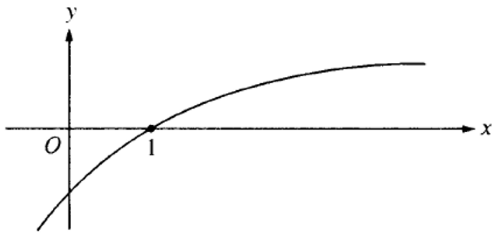
$- \quad 0 \quad +$

If $f''(x) = x(x+1)(x-2)^2$, then the graph of f has inflection points when $x =$

(A) -1 only (B) 2 only $\boxed{\text{(C) } -1 \text{ and } 0 \text{ only}}$ (D) -1 and 2 only (E) -1, 0, and 2 only

$f''(-2) \quad f''(-\frac{1}{2}) \quad f''(1) \quad f''(3)$

$+ \quad -1 \quad - \quad 0 \quad + \quad 2 \quad +$



The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

(A) $f(1) < f'(1) < f''(1)$

(B) $f(1) < f''(1) < f'(1)$

(C) $f'(1) < f(1) < f''(1)$

(D) $\boxed{f''(1) < f(1) < f'(1)}$

(E) $f''(1) < f'(1) < f(1)$

$f(1) = 0$

$f'(1) > 0$

$f''(1) < 0$

A function f is continuous at $x = c$ if $\lim_{x \to c} f(x) = f(c)$.

<can draw graph w/o picking up pencil>

f doesn't have any holes, jumps, or vertical asymptotes.

A function f is differentiable at $x=c$ if $\lim\limits_{x \to c} \frac{f(x)-f(c)}{x-c} = L$ for some real number L.

(or if $\lim\limits_{h \to 0} \frac{f(c+h)-f(c)}{h} = L$)

<slope of f from left & right is the same>

no sharp points, cusps or vertical tangent lines

Let f be a function that is differentiable on the open interval $(1,10)$. If $f(2)=-5$, $f(5)=5$, and $f(9)=-5$, which of the following must be true?

f is also cts. on $(1,10)$

Differentiability implies continuity

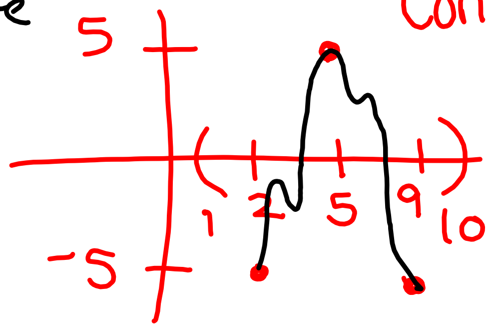
I. f has at least 2 zeros. ✓

II. The graph of f has at least one horizontal tangent. ✓

III. For some c, $2<c<5$, $f(c)=3$. ✓

Intermediate Value Theorem

(A) None
(B) I only
(C) I and II only
(D) I and III only
(E) I, II, and III



A spherical balloon is inflated with gas at the rate of 300 cubic centimeters per minute. How fast is the radius of the balloon increasing at the instant the radius is 70 centimeters?

a. $\frac{dr}{dt} = \frac{3}{98\pi}$ cm/min

b. $\frac{dr}{dt} = \frac{1}{98\pi}$ cm/min

c. $\boxed{\frac{dr}{dt} = \frac{3}{196\pi} \text{ cm/min}}$

d. $\frac{dr}{dt} = 98\pi$ cm/min

e. $\frac{dr}{dt} = 196\pi$ cm/min

$V = \frac{4}{3}\pi r^3$ $\quad \frac{dV}{dt} = 300 \frac{cm^3}{min}$

$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$ $\quad \frac{dr}{dt} = ?$ when $r = 70$ cm

$\frac{\frac{dV}{dt}}{4\pi r^2} = \frac{dr}{dt}$ $\quad = \frac{300}{4\pi(70)^2} = \frac{300}{4\pi \cdot 49 \cdot 100}$

A conical tank (with vertex down) is 12 feet across the top and 18 feet deep. If water is flowing into the tank at a rate of 18 cubic feet per minute, find the rate of change of the depth of the water when the water is 10 feet deep.

a. $\frac{9}{40\pi}$ ft/min

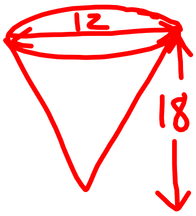
b. $\frac{9}{100\pi}$ ft/min

c. $\frac{81}{20\pi}$ ft/min

d. $\boxed{\frac{81}{50\pi} \text{ ft/min}}$

e. $\frac{81}{200\pi}$ ft/min

$V = \frac{1}{3}\pi r^2 h$ $\quad \frac{dV}{dt} = 18 \frac{ft^3}{min}$; $\frac{dh}{dt} = ?$ when $h = 10$ ft



$\frac{12}{18} = \frac{2r}{h} \rightarrow r = \frac{h}{3}$

$V = \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 \cdot h$

$V = \frac{\pi}{27} h^3$

$\frac{dV}{dt} = \frac{\pi}{9} h^2 \cdot \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{\frac{dV}{dt}}{\frac{\pi}{9} h^2}$

$= \frac{18}{\frac{\pi}{9}(10)^2} = \frac{\cancel{18}^{9} \cdot 9}{\pi \cdot \cancel{100}_{50}} = \frac{81}{50\pi}$

Locate the absolute extrema of the function $g(t) = \dfrac{t^2}{t^2+2}$ on the closed interval $[-3, 3]$.

a. The absolute maximum is $\frac{9}{11}$, and it occurs at the critical number $x = 0$.
The absolute minimum is $\frac{9}{5}$, and it occurs at the left endpoint $x = -3$.

b. The absolute maximum is $\frac{9}{11}$, and it occurs at either endpoint $x = \pm 3$.
The absolute minimum is 0, and it occurs at the critical number $x = 0$.

c. The absolute maximum is $\frac{9}{11}$, and it occurs only at the left endpoint $x = -3$.
The absolute minimum is 0 and it occurs at the critical number $x = 0$.

d. The absolute maximum is $\frac{9}{11}$, and it occurs at the critical number $x = 0$.
The absolute minimum is $\frac{9}{5}$, and it occurs at the right endpoint $x = 3$.

e. The absolute maximum is $\frac{9}{11}$, and it occurs only at the right endpoint $x = 3$
The absolute minimum is 0 and it occurs at the critical number $x = 0$.

$f(-3) = \frac{9}{11}$

$f(0) = 0$

$f(3) = \frac{9}{11}$

Determine whether Rolle's Theorem can be applied to the function $f(x) = x^2 - 2x - 3$ on the closed interval $[-1,3]$. If Rolle's Theorem can be applied, find all values of c in the open interval $(-1,3)$ such that $f'(c) = 0$.

a. Rolle's Theorem applies; $c = 1$
b. Rolle's Theorem applies; c = 2
c. Rolle's Theorem applies; $c = 0$
d. Rolle's Theorem applies; $c = -1$
e. Rolle's Theorem does not apply

$f(x) = (x-3)(x+1)$

$f(-1) = 0$
$f(3) = 0$

$f'(x) = 2x-2$

$2x-2=0$
$2x=2$
$x=1$

Determine whether the Mean Value Theorem can be applied to the function $f(x) = x^2$ on the closed interval [3,9]. If the Mean Value Theorem can be applied, find all numbers c in the open interval (3,9) such that $f'(c) = \dfrac{f(9) - f(3)}{9 - (3)}$.

$f'(x) = 2x$

a. MVT applies; $c = 6$
b. MVT applies; $c = 7$
c. MVT applies; $c = 4$
d. MVT applies; c = 5
e. MVT applies; $c = 8$

$$\frac{f(9)-f(3)}{9-3} = \frac{9^2-3^2}{6} = \frac{81-9}{3\cdot 2}$$

$$= \frac{27-3}{2} = \frac{24}{2}$$

$$= 12$$

$2x = 12$

$x = 6$

3.7 Optimization Problems

4. Find two positive numbers (x, y) whose product is 192 and the sum of the first plus three times the second is a minimum.

want to minimize sum function S

SUM	is	minimum

$xy = 192 \longrightarrow y = \frac{192}{x}$

$S(x,y) = x + 3y$

$S(x) = x + 3\left(\frac{192}{x}\right) = x + 3(192)x^{-1}$

$S'(x) = 1 + 3(192)(-x^{-2})$

$0 = 1 - \frac{3(192)}{x^2}$

$\frac{3(192)}{x^2} = 1$

$3(192) = x^2$

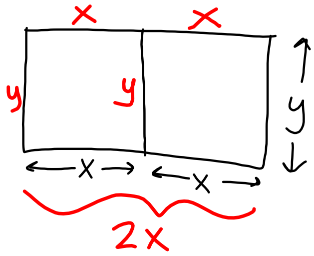
$\sqrt{3(192)} = x$

$\boxed{24 = x}$

$y = \frac{192}{24} =$

$\boxed{y = 8}$

18. A rancher has 200 feet of fencing with which to enclose two adjacent corrals, arranged according to the figure. What dimensions should be used so that the enclosed area will be a maximum?



trying to maximize Area

$A(x,y) = 2xy$

$200 = 4x + 3y$

$200 - 3y = 4x$

$50 - \frac{3}{4}y = x$

$A(y) = 2\left(50 - \frac{3}{4}y\right)y$

$A(y) = 100y - \frac{3}{2}y^2$

$A'(y) = 100 - 3y$

$100 - 3y = 0$

$100 = 3y$

$\boxed{\frac{100}{3}\text{ft} = y}$

$x = 50 - \frac{3}{4}\left(\frac{100}{3}\right)$

$\boxed{x = 25\text{ft}}$

8.7 Indeterminate Forms & L'Hôpital's Rule

$\boxed{\frac{0}{0},\ \frac{\infty}{\infty},}$ $0 \cdot \infty,\ 1^{\infty},\ 0^0,\ \textit{and}\ \infty - \infty$ are called indeterminate forms.

L'Hôpital's Rule:

Let f and g be functions that are differentiable on an open interval (a, b) containing c, except possibly at c itself. Assume that $g'(x) \neq 0$ for all x in (a, b), except possibly at c itself. If the limit of f(x)/g(x) as x approaches c produces an indeterminate form $0/0$, ∞/∞, $(-\infty)/\infty$, or $\infty/(-\infty)$, then

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

20. $$\lim_{x\to 0} \frac{\sin ax}{\sin bx} = \lim_{x\to 0} \frac{\cos(ax)\cdot a}{\cos(bx)\cdot b}$$

$$= \frac{\cos(a\cdot 0)\cdot a}{\cos(b\cdot 0)\cdot b}$$

$$= \frac{a}{b}$$