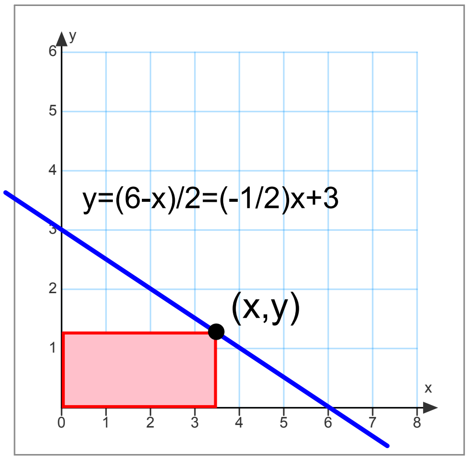
Assignments for the week of 10/24:

- Read 3.5,3.7,7.7
- 45 minutes of Khan Academy
- Textbook assignment due Tuesday, 11/01 (final exam day):
  3.5 #15-31odd - limits at infinity
  3.7 #3,5,17,19,23 - optimization
  8.7 #11-35odd - l'Hopital's rule
  ~~8.7 #47-55odd - l'Hopital's rule with logs~~

FINAL EXAM - Tuesday, 11/01 - Comprehensive; can replace lowest test grade

REVIEW SESSION - Monday @ 3pm?

24. A rectangle is bounded by the x- and y-axes and the graph of y=(6-x)/2. What length and width should the rectangle have so that its area is a maximum?



$A(x,y) = xy$

$y = \frac{6-x}{2} = -\frac{1}{2}x + 3$

$A(x) = x\left(-\frac{1}{2}x + 3\right)$

$= -\frac{1}{2}x^2 + 3x$

$A'(x) = -x + 3$

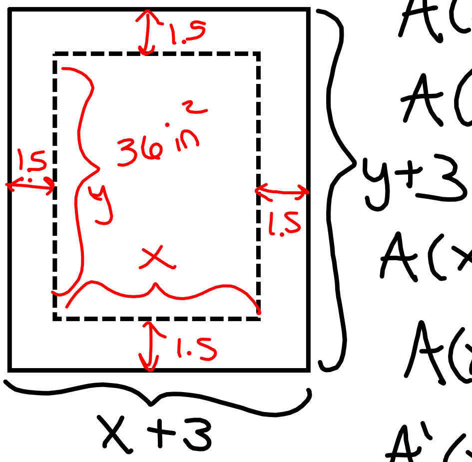
$-x + 3 = 0$

$\boxed{\text{width: } x = 3}$

$y = \frac{6-x}{2} = \frac{6-3}{2}$

$\boxed{y = \frac{3}{2}}$

30. A rectangular page is to contain 36 square inches of print. The margins on each side are to be 1.5 inches. Find the dimensions of the page such that the least amount of paper is used.



$A(x,y) = (x+3)(y+3) \quad ; \quad xy = 36$

$A(x) = (x+3)\left(\frac{36}{x}+3\right) \quad y = \frac{36}{x}$

$A(x) = 36 + 3x + \frac{3(36)}{x} + 9$

$A(x) = 45 + 3x + 3(36)x^{-1}$

$A'(x) = 3 - 3(36)x^{-2}$

$3 - \frac{3(36)}{x^2} = 0$

$\frac{x^2}{\cancel{3}} \cdot \cancel{3} = \frac{\cancel{3}(36)}{\cancel{x^2}} \cdot \frac{\cancel{x^2}}{\cancel{3}}$

$x^2 = 36$

$x = 6$

$y = 6$

$\boxed{\text{dimensions of page } 9\text{in} \times 9\text{in}}$

8.7 Indeterminate Forms & L'Hôpital's Rule

$\boxed{\frac{0}{0}, \ \frac{\infty}{\infty},}$ $0 \cdot \infty, \ 1^{\infty}, \ 0^0, \ and \ \infty - \infty$ are called <u>indeterminate forms</u>.

<u>L'Hôpital's Rule</u>:

Let f and g be functions that are differentiable on an open interval (a, b) containing c, except possibly at c itself. Assume that $g'(x) \neq 0$ for all x in (a, b), except possibly at c itself. If the limit of f(x)/g(x) as x approaches c produces an indeterminate form $0/0$, ∞/∞, $(-\infty)/\infty$, or $\infty/(-\infty)$, then

$$\lim_{x\to c}\frac{f(x)}{g(x)} = \lim_{x\to c}\frac{f'(x)}{g'(x)}$$

7.7

12. $\lim_{x\to -1} \frac{x^2-x-2}{x+1} = \lim_{x\to -1} \frac{(x-2)\cancel{(x+1)}}{\cancel{x+1}} = -1-2 = \boxed{-3}$

$= \lim_{x\to -1} \frac{2x-1}{1} = 2(-1)-1 = \boxed{-3}$

16. $\lim_{x\to 0^+} \frac{e^x-(1+x)}{x^3} = \frac{0}{0}$

$= \lim_{x\to 0^+} \frac{e^x-1}{3x^2} = \frac{0}{0}$

$= \lim_{x\to 0^+} \frac{e^x}{6x} = \boxed{+\infty}$

18. $$\lim_{x\to 1} \frac{\ln x^2}{x^2-1} = \frac{0}{0}$$

$$= \lim_{x\to 1} \frac{\frac{1}{x^2}\cdot \cancel{2x}}{\cancel{2x}}$$

$$= \frac{1}{1^2} = \boxed{1}$$

20. $$\lim_{x\to 0} \frac{\sin ax}{\sin bx} = \lim_{x\to 0} \frac{\cos(ax)\cdot a}{\cos(bx)\cdot b}$$

$$= \frac{\cos(a\cdot 0)\cdot a}{\cos(b\cdot 0)\cdot b}$$

$$= \frac{a}{b}$$

28. $\lim_{x \to \infty} \frac{x^2}{e^x} = \frac{\infty}{\infty}$

$= \lim_{x \to \infty} \frac{2x}{e^x} = \frac{\infty}{\infty}$

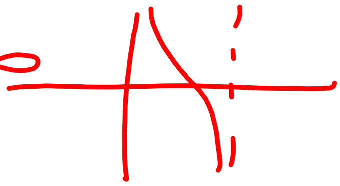
$= \lim_{x \to \infty} \frac{2}{e^x} = \boxed{0}$

36. $\lim_{x \to \infty} \frac{e^{x/2}}{x} = \frac{\infty}{\infty}$

$\frac{x}{2} = \frac{1}{2}x$

$= \lim_{x \to \infty} \frac{e^{x/2} \cdot \frac{1}{2}}{1} = \boxed{\infty}$

38. $\lim_{x\to 0^+} x^3 \cot x = 0\cdot\infty$



$$= \lim_{x\to 0^+} \frac{x^3}{\tan x} = \frac{0}{0}$$

$$= \lim_{x\to 0^+} \frac{3x^2}{\sec^2 x} = \frac{0}{1} = \boxed{0}$$

40. $\lim_{x\to\infty} x\tan\frac{1}{x} = \infty\cdot 0$

$$= \lim_{x\to\infty} \frac{\tan\frac{1}{x}}{\frac{1}{x}} = \frac{0}{0}$$

$$= \lim_{x\to\infty} \frac{\sec^2\frac{1}{x}\cdot\cancel{-\frac{1}{x^2}}}{\cancel{-\frac{1}{x^2}}} = \boxed{1}$$

44. $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = y$

$\ln\left[\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x\right] = \ln y$

$\lim_{x\to\infty} \ln\left(1+\frac{1}{x}\right)^{x} = \ln y$

$\lim_{x\to\infty} x \ln\left(1+\frac{1}{x}\right) = \ln y$ $(\infty \cdot 0)$

$\lim_{x\to\infty} \frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}} = \ln y$

$\lim_{x\to\infty} \frac{\frac{1}{1+\frac{1}{x}} \cdot \left(\cancel{\frac{-1}{x^2}}\right)}{\cancel{\frac{-1}{x^2}}} = \ln y$

$1 = \ln y$

$e^1 = e^{\ln y}$

$\boxed{e = y}$

$\frac{d^2y}{dx^2} = y''$ $x^2 = y^3$

$$\frac{-1}{x^2} \le \frac{\sin[f(x)]}{x^2} \le \frac{1}{x^2}$$

$\frac{-1}{x^2} \to 0$, $\frac{1}{x^2} \to 0$

$$f(x) = x^3 - 12x$$

$$f'(x) = \lim_{x \to 0} \frac{(x+h)^3 - 12(x+h) - [x^3 - 12x]}{h}$$