Topics that may appear on the Final Exam:

- calculate limits (by plugging in, rewriting, applying l'hopital's rule, squeeze theorem, using knowledge of graphs, etc.)
- limit definition of the derivative and continuity
- statement of epsilon-delta definition of limit
- determining the graph corresponding to statements about continuity, 1st and/or 2nd derivatives
- determining continuity, first and/or second derivatives of a given function (including piecewise)
- Intermediate Value Theorem
- Mean Value Theorem (Rolle's Theorem)
- differentiation (know basic rules, including power, product, quotient, chain, logs, exp, and trig)
- implicit differentiation
- equation of a tangent line
- average and instantaneous rates of change
- related rates ; optimization
- relative extrema on the domain of a function
- absolute extrema on a closed interval [a,b]
- determining where a function in incresing/decreasing and/or concave up/down

If $\lim_{x \to c} \frac{f(x)}{g(x)}$ yields $\frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$

then $\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$.

If $g(x) \le f(x) \le h(x)$

and $\lim_{x \to c} g(x) = L = \lim_{x \to c} h(x)$,

then $\lim_{x \to c} f(x) = L$.

If f is continuous on $[a, b]$

and $f(a) < L < f(b)$

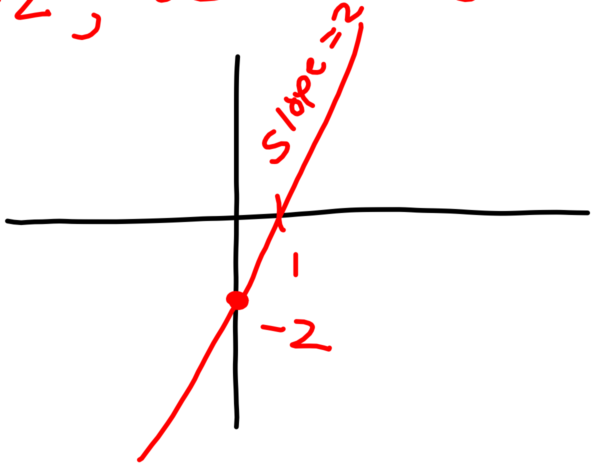
or $f(b) < L < f(a)$, then

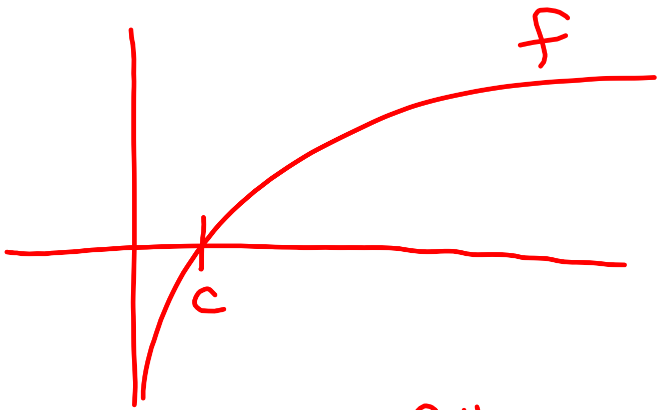
there exist $c \in (a, b)$ such

that $f(c) = L$.

$f(0) = -2$

$f'(x) = 2, -\infty < x < \infty$





$f(c) = 0$ $f''(c) < 0$ b/c f is concave down

$f'(c) > 0$ b/c f is increasing

$s(t) = \frac{1}{2}at^2 + v_0 t + s_0$

average velocity = avg r.o.c of position = $\frac{s(b) - s(a)}{b-a}$

inst. velocity = inst. r.o.c of position = $s'(c)$

inst. acc = $s''(c) = v'(c)$

avg acc = $\frac{v(b) - v(a)}{b-a}$

6. Find an equation of the tangent line to the graph of f at the indicated point.

$f(x) = \sqrt{2x^2 - 7}$, $(2, 1)$

$f(x) = (2x^2 - 7)^{1/2}$

$f'(x) = \frac{1}{2}(2x^2 - 7)^{-1/2} \cdot 4x$

$= \frac{2x}{\sqrt{2x^2 - 7}}$

$m = \frac{2(2)}{\sqrt{2(2)^2 - 7}} = 4$

$y - y_1 = m(x - x_1)$

$y - 1 = 4(x - 2)$

$y - 1 = 4x - 8$

$\boxed{y = 4x - 7}$

$4x - y = 7$

8. Find $\frac{dy}{dx}$ by implicit differentiation and evaluate the derivative at the indicated point.

a. $x^{2/3} + y^{2/3} = 5,\ (8, 1)$

b. $x\cos y = 1,\ \left(2, \frac{\pi}{3}\right)$

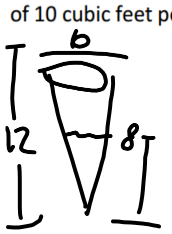
9. Find $\frac{d^2y}{dx^2}$ $= y''$ in terms of x and y.

$$\frac{d^{(n)}y}{dx^n} = y^{(n)}$$

a. $x^2 = y^3$

b. $1 - xy = x - y$

11. A conical tank (with vertex down) is 6 feet across and 12 feet deep. If water is flowing into the tank at a rate of 10 cubic feet per minute, find the rate of change of the depth of the water when the water is 8 feet deep.



$\frac{dV}{dt} = 10\,\frac{ft^3}{m}$ $\quad V = \frac{1}{3}\pi r^2 h$

$V = \frac{\pi}{3}\left(\frac{h}{4}\right)^2 h$

3:12

1:4

$\frac{dV}{dt}$

$V = \frac{\pi}{48} h^3$

$$\frac{dV}{dt} = \frac{3\pi}{48} h^2 \cdot \frac{dh}{dt}$$

$$\frac{\pi}{16} h^2 \cdot \frac{dh}{dt}$$

$$\frac{10\,\frac{ft^3}{min}}{\frac{\pi}{16} h^2} = \frac{dh}{dt}$$

$$\frac{10\,\frac{ft^3}{min}}{\frac{\pi}{16}\cdot 8^2} = \frac{10\,\frac{ft^3}{min}}{\pi\,\frac{64}{16}} \quad 4 \quad \frac{5}{2\pi}\,\frac{ft}{min}$$

13. Find the limit.

$a. \lim_{x\to 0} \frac{\sin x\ (1-\cos x)}{2x^2} = 0$

$b. \lim_{x\to 3^-} f(x)$, where $f(x) = \begin{cases} \frac{x+2}{2}, x \le 3 \\ \frac{12-2x}{3}, x > 3 \end{cases} = \frac{3+2}{2} = \frac{5}{2}$

$c. \lim_{x\to 3^+} \frac{x^2}{x^2-9} = \lim_{x\to 3^+} \frac{x^2}{(x-3)(x+3)} = \infty$

$d. \lim_{x\to\infty} \frac{3x^3+2}{9x^3-2x^2+7} = \frac{1}{3}$

(a) $\lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{1-\cos x}{x} \cdot \frac{1}{2}$

$\downarrow \quad \downarrow \quad \downarrow$

$1 \cdot 0 \cdot \frac{1}{2}$

$e. \lim_{x\to -\infty} \frac{3x-5}{\sqrt{4x^2+2x-1}} = \lim_{x\to-\infty} \frac{3x}{\sqrt{4x^2}} = \lim_{x\to-\infty} \frac{3x}{2|x|} = \lim_{x\to-\infty} \frac{3x}{-2x} = \boxed{\frac{-3}{2}}$

$f. \lim_{x\to 1} \frac{\ln x^2}{x^2-1} = \lim_{x\to 1} \frac{\frac{1}{x^2} \cdot \cancel{2x}}{\cancel{2x}} = \boxed{1}$

$g. \lim_{x\to\infty} \frac{e^{x/2}}{x^3} = \lim_{x\to\infty} \frac{e^{x/2} \cdot \frac{1}{2}}{3x^2} = \lim_{x\to\infty} \frac{e^{x/2} \cdot \frac{1}{2} \cdot \frac{1}{2}}{6x} = \lim_{x\to\infty} \frac{e^{x/2} \cdot \frac{1}{2} \cdot \frac{1}{4}}{6} = \boxed{\infty}$

$h. \lim_{x\to\infty} \frac{\sin x}{x} = 0$

$i. \lim_{x\to\infty} (1+x)^{1/x} = e$

y not on final

ln of both sides

$-1 \le \sin x \le 1$

$\frac{-1}{x} \le \frac{\sin x}{x} \le \frac{1}{x}$

$\lim_{x\to\infty} \frac{-1}{x} \le \lim_{x\to\infty} \frac{\sin x}{x} \le \lim_{x\to\infty} \frac{1}{x}$

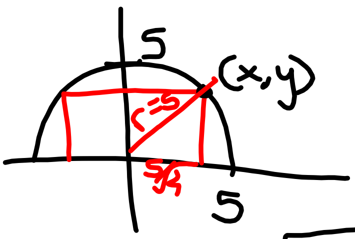
$0 \le \lim_{x\to\infty} \frac{\sin x}{x} \le 0$

14. Find the derivative of the function using the definition (limit of the difference quotient)
$f(x) = x^3 - 12x$

$x = c$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^3 - 12(x+h) - (x^3 - 12x)}{h} = \lim_{x \to c} \frac{x^3 - 12x - (c^3 \ldots}{x - c}$$

15. A rectangle is bounded by the x-axis and the semi-circle $y = \sqrt{25 - x^2}$ (see figure on p. 217 of textbook). What length and width should the rectangle have so that its area is a maximum?



$$A = 2xy = 2x\sqrt{25 - x^2} = 2x(25 - x^2)^{1/2}$$

$$A' = 2x \cdot \tfrac{1}{2}(25 - x^2)^{-1/2} \cdot (-2x) + 2(25 - x^2)^{1/2}$$

$$= -2x^2(25 - x^2)^{-1/2} + 2(25 - x^2)^{1/2} \stackrel{\text{set}}{=} 0$$

$$\sqrt{25 - x^2}\left(2\sqrt{25 - x^2}\right) = \left(2x^2 / \sqrt{25 - x^2}\right)\sqrt{25 - x^2}$$

$$\frac{2(25 - x^2)}{2} = \frac{2x^2}{2} \quad 25 - x^2 = x^2 \quad +x^2$$

$$\frac{25}{2} = \frac{2x^2}{2}$$

$$\frac{5}{\sqrt{2}} = x$$

$$l = \frac{2 \cdot 5}{\sqrt{2}} = \boxed{\frac{10}{\sqrt{2}}}$$

$$h = \sqrt{25 - \left(\frac{5}{\sqrt{2}}\right)^2} = \boxed{\frac{5}{\sqrt{2}}}$$