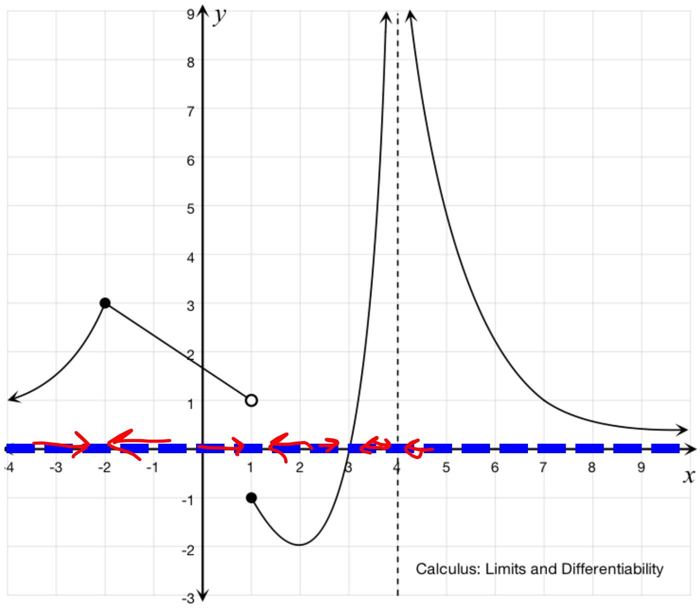
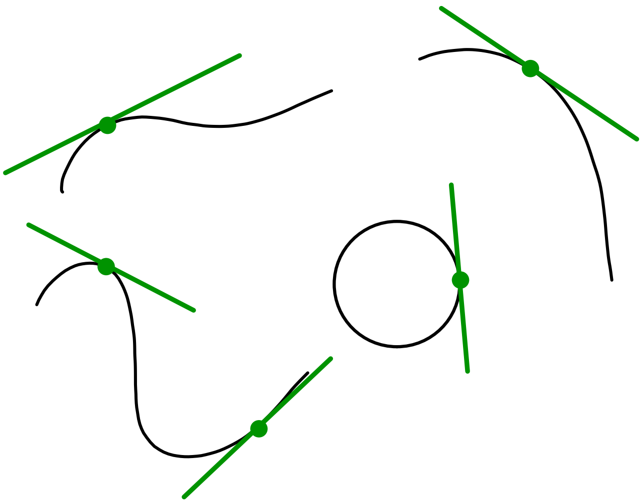
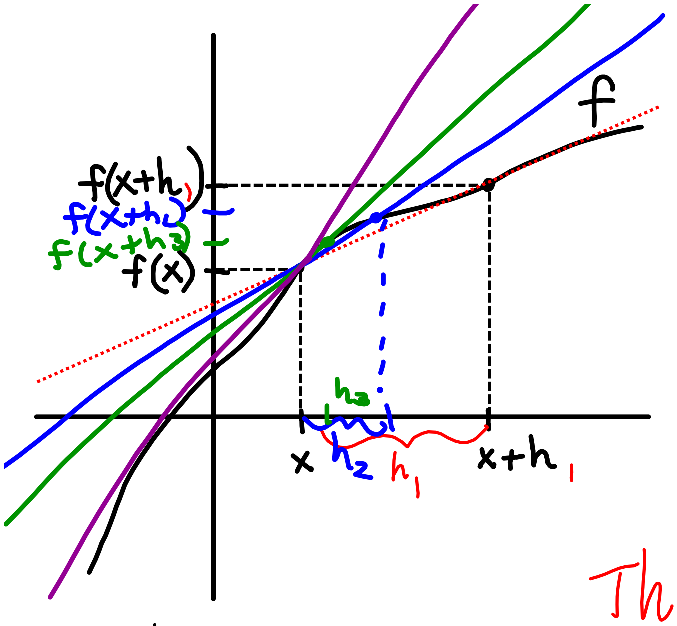


as ***x*** approaches..	***f(x)*** approaches...
-2	3
1^- (from the left)	1
1^+ (from the right)	-1
3	0
$-\infty$	0
∞	0
4	∞

tangent lines





secant line

$$\text{slope} = \frac{\Delta y}{\Delta x}$$

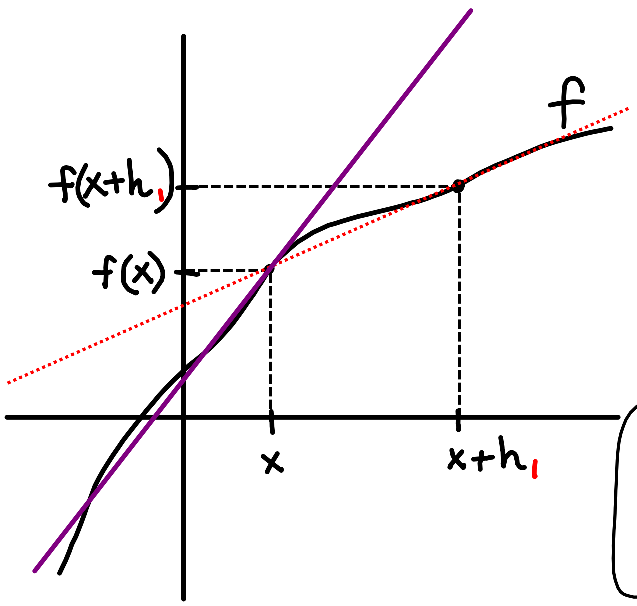
$$m = \frac{f(x+h) - f(x)}{x+h-x}$$

$$= \frac{f(x+h) - f(x)}{h}$$

The Difference Quotient

equation:

$$y - y_1 = m(x - x_1)$$



tangent line

Let the distance h away from x get smaller & smaller (let it approach zero)

$$\boxed{\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

= slope of tangent line @ $(x, f(x))$

Δx "delta x"
means change in x

$$\frac{f(x+\Delta x)-f(x)}{\underbrace{\Delta x}} = \frac{f(x+h)-f(x)}{h}$$

treated as a single variable

1.2

$$\lim_{x\to 2} f(x) = \frac{1}{4}$$

$$f(x)=\frac{x-2}{x^2-4}, \quad x\neq 2,-2$$

What happens to *f(x)* as *x* approaches 2?

x	1.9	1.99	1.999	2	2.001	2.01	2.1
f(x)	0.2564	0.2506	0.2501	0.25 $=\frac{1}{4}$	0.2499	0.2493	0.2439

Informal Description of the Limit

If *f(x)* becomes arbitrarily close to a single REAL number *L* as *x* approaches *c* from either side, the limit of *f(x)*, as *x* approaches *c*, is *L*.

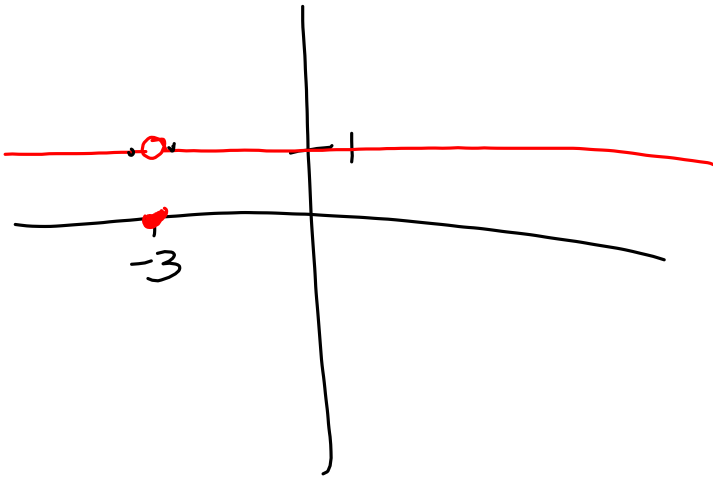
$$\lim_{x \to c} f(x) = L$$

Note: the existence or nonexistence of *f(x)* at *x=c* has no bearing on the existence of the limit as *x* approaches *c*.

A function can be undefined for a certain value of *c* with the limit as *x* approaches *c* still defined.

$$\lim_{x \to -3} \frac{\sqrt{1-x} - 2}{x+3} = -0.25$$

$$f(x)=\begin{cases}1, & x\neq -3\\ 0, & x=-3\end{cases}$$



$$\lim_{x\to -3} f(x) = \boxed{1}$$

1.2

1-6

15-24