**Informal Description of the Limit**

If *f(x)* becomes arbitrarily close to a single REAL number *L* as *x* approaches *c* from either side, the limit of *f(x)*, as *x* approaches *c*, is *L*.

$$\lim_{x \to c} f(x) = L$$

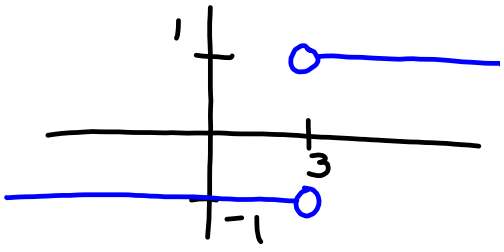
Note: the existence or nonexistence of *f(x)* at *x=c* has no bearing on the existence of the limit as *x* approaches *c*.

$$\lim_{x \to 3} \frac{|x-3|}{x-3} \quad \text{DNE}$$

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$|-2| = 2 = -(-2)$

$$\frac{|x-3|}{x-3} = \begin{cases} \frac{x-3}{x-3} = 1, & x-3 > 0 \\ & x > 3 \\ \frac{-(x-3)}{x-3} = -1, & x-3 < 0 \\ & x < 3 \end{cases}$$

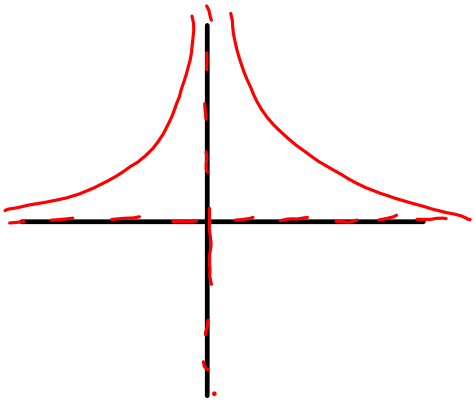


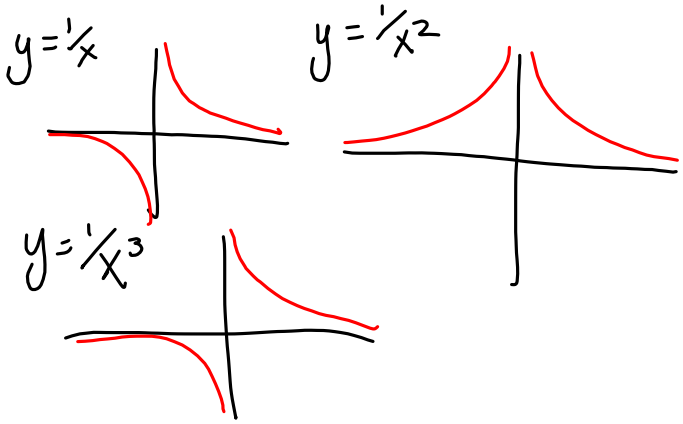

$$\lim_{x \to 3^-} \frac{|x-3|}{x-3} = -1 \; ; \; \lim_{x \to 3^+} \frac{|x-3|}{x-3} = 1$$

$$\lim_{x \to 0} \frac{1}{x^4} = \boxed{\infty}$$

$\frac{1}{1/10}$, $\frac{1}{1/100}$, $\frac{1}{1/1000}$, $\frac{1}{1/1000000}$

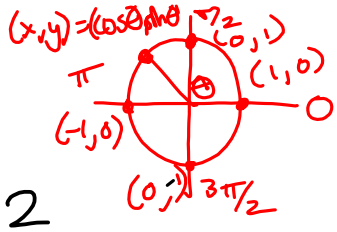
10, 100, 1000, 1000000






$$\lim_{x \to 0} \sin\frac{1}{x} = \boxed{\text{DNE}}$$

$\sin \frac{1}{2/\pi}$

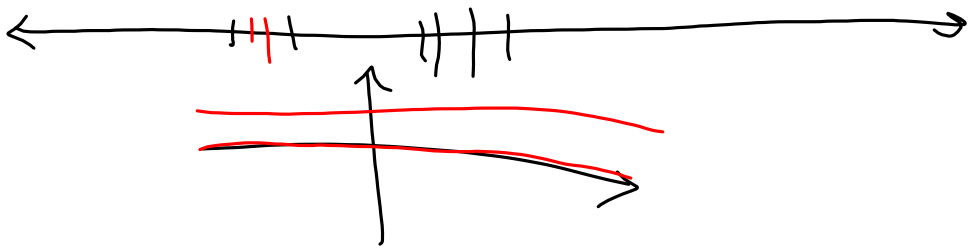



| $x$ | $\frac{2}{\pi}$ | $\frac{2}{3\pi}$ | $\frac{2}{5\pi}$ | $\frac{2}{7\pi}$ | $\frac{2}{9\pi}$ | $\frac{2}{11\pi}$ |
|---|---|---|---|---|---|---|
| $\sin\frac{1}{x}$ | $\frac{1}{x} = \frac{\pi}{2}$<br>$\sin\frac{\pi}{2}$<br>1 | $\sin\frac{3\pi}{2}$<br>$-1$ | $\sin\frac{5\pi}{2}$<br>1 | $\sin\frac{7\pi}{2}$<br>$-1$ | 1 | $-1$ |

"Dirichlet Function"

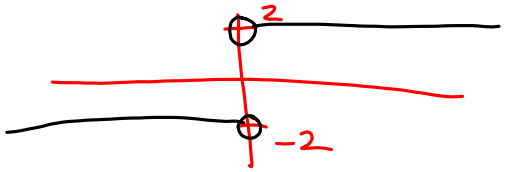
$$f(x)=\begin{cases}0, & \text{if } x \text{ is rational}\\ 1, & \text{if } x \text{ is irrational}\end{cases}$$

limits don't exist anywhere



$$\lim_{x\to 0}\frac{|2x|}{x}$$

$$\frac{|2x|}{x}=\begin{cases}\frac{2x}{x}=2, & 2x>0,\ x>0\\ \frac{-2x}{x}=-2, & 2x<0,\ x<0\end{cases}$$




$$\lim_{x\to 0^+} f(x)=2$$

$$\lim_{x\to 0^-} f(x)=-2$$

$$\lim_{x\to 0} f(x)=\text{DNE}$$

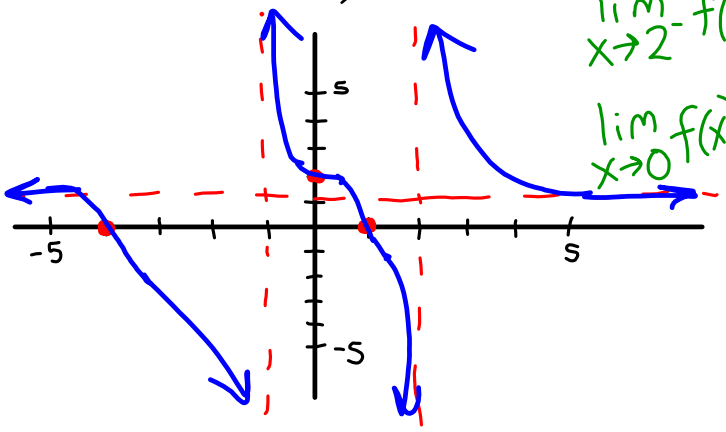
Graph the rational function.

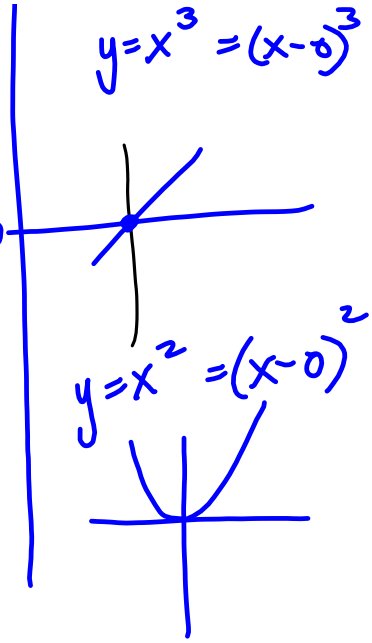
$f(x) = \dfrac{(x+4)(x-1)}{(x-2)(x+1)} \approx \dfrac{x^2}{x^2} = 1$

$\lim_{x\to\infty} f(x) = 1$

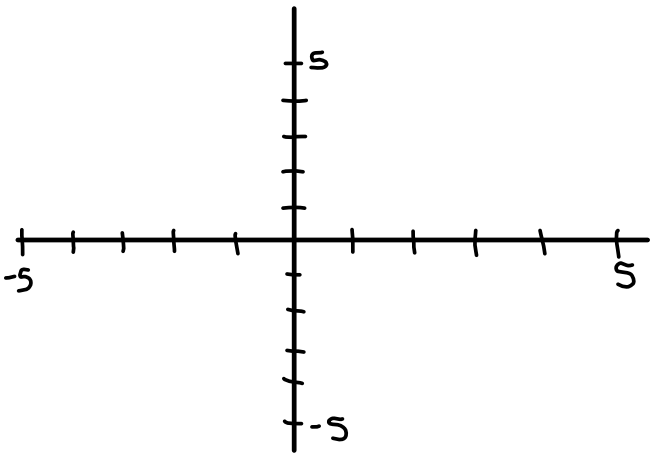
$\lim_{x\to 2^-} f(x) = -\infty$

$\lim_{x\to 0} f(x) = 2$




$f(x) = \dfrac{x(x-2)}{x+3}$




$\lim_{x\to-\infty} f(x) =$

$\lim_{x\to-3^+} f(x) =$

$\lim_{x\to 2} f(x) =$

$\varepsilon - \delta$