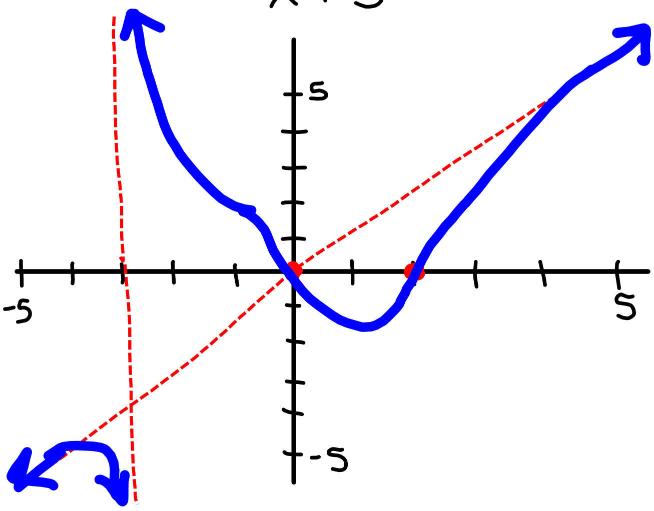
$$f(x) = \frac{x(x-2)}{x+3} \approx \frac{x^2}{x} = x$$



$$\lim_{x\to -\infty} f(x) = -\infty$$

$$\lim_{x\to -3^+} f(x) = \infty$$

$$\lim_{x\to 2} f(x) = 0$$

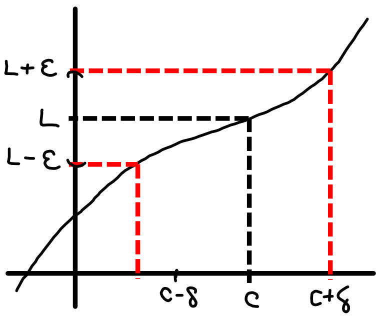
Building up to the $\epsilon - \delta$ Definition of the Limit

ε = epsilon

δ = delta

Translating the "informal description": $\lim_{x\to c} f(x) = L$

If $f(x)$ becomes arbitrarily close to a single number L as x approaches c from either side, the limit of $f(x)$, as x approaches c, is L.



"$f(x)$ becomes arbitrarily close to L"

$f(x)$ lies in the interval $(L-\varepsilon, L+\varepsilon)$ for some (really small) $\varepsilon > 0$.

$|f(x) - L| < \varepsilon$

"the distance between $f(x)$ and L is less than ε"

"x approaches c"

There exists a (very small) positive number δ such that x is either in the interval $(c-\delta, c)$ or $(c, c+\delta)$.

$0 < |x - c| < \delta$

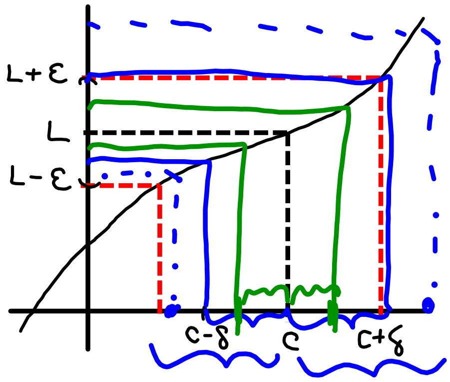
The first inequality guarantees that $x \neq c$.

$\varepsilon - \delta$ Definition of the Limit:

Let f be a function defined on an open interval containing c (except possibly at c) and let L be a real number. The statement

$$\lim_{x \to c} f(x) = L$$

means that for each $\varepsilon > 0$, there exists a $\delta > 0$ such that if $0 < |x - c| < \delta$, then $|f(x) - L| < \varepsilon$.

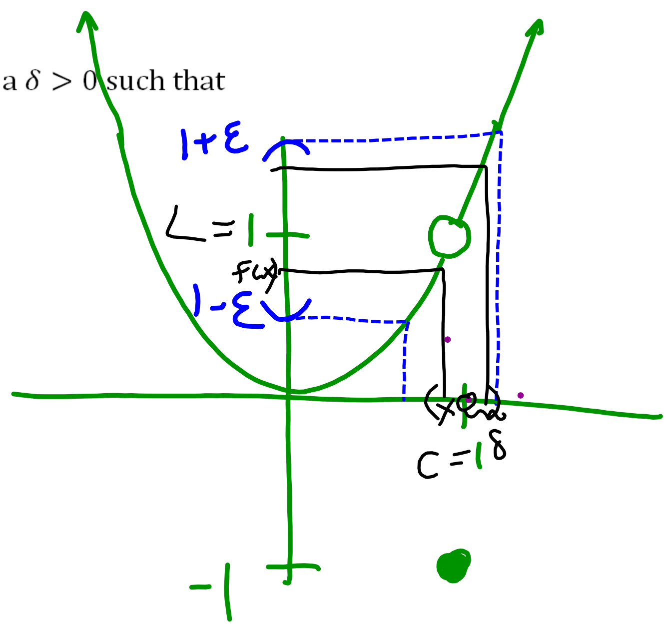


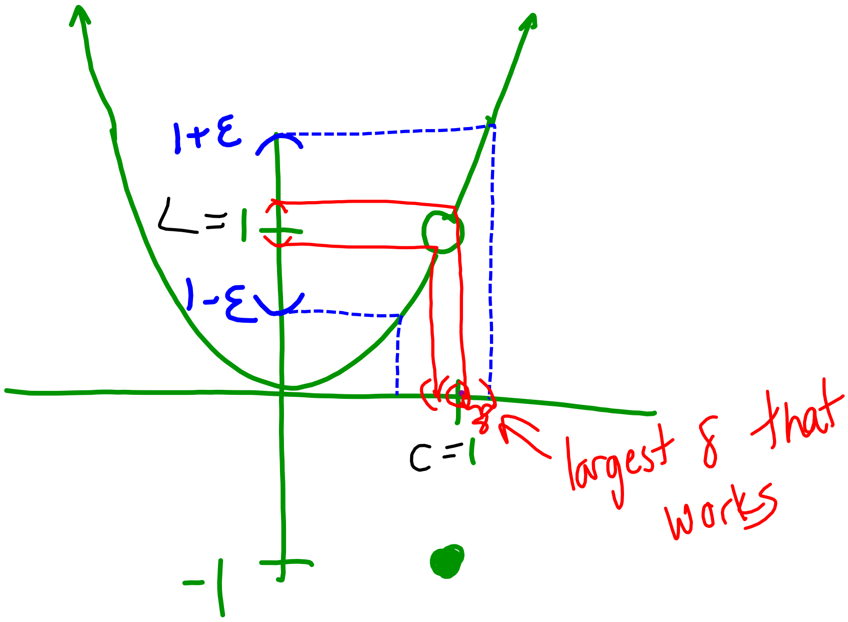
$\varepsilon - \delta$ Definition of the Limit:

$\lim_{x \to c} f(x) = L$ if given $\varepsilon > 0$, there exists a $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.

$$f(x) = \begin{cases} x^2, & x \neq 1 \\ -1, & x = 1 \end{cases}$$

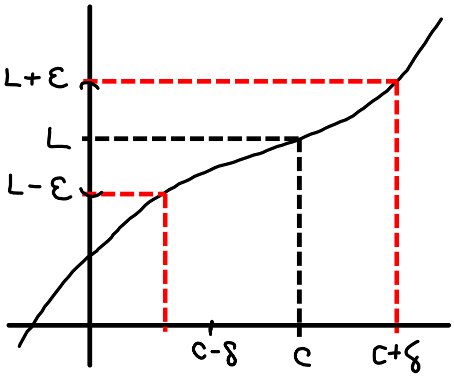
$$\lim_{x \to 1} f(x) = 1$$





$\varepsilon - \delta$ Definition of the Limit:

$\lim_{x \to c} f(x) = L$ if given $\varepsilon > 0$, there exists a $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.



If a, then b.

$a \Rightarrow b$

$|x-c|<\delta \Rightarrow |f(x)-L|<\varepsilon$

$\varepsilon - \delta$ Definition of the Limit:

$\lim_{x \to c} f(x) = L$ if given $\varepsilon > 0$, there exists a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.

$f(x) = 2x - 1$, $L = 7$, $c = 4$

Find $\lim_{x \to 4} f(x)$ and prove that is the limit using the ε-δ definition.

$L = 2(4) - 1 = 7$

Let $\varepsilon > 0$ be given. We want to find a $\delta > 0$ such that $|f(x) - 7| < \varepsilon$ whenever $|x-4| < \delta$.

$|f(x) - 7| = |2x - 1 - 7| = |2x - 8| = |2(x-4)| =$

$|f(x) - 7| = 2|x-4| < \varepsilon$

$|x - 4| < \boxed{\varepsilon/2} = \delta$

Take $\delta = \varepsilon/2$. Then whenever $|x-4| < \delta$, we have that

$|f(x) - 7| = 2|x-4| < 2\delta = 2 \cdot \frac{\varepsilon}{2} = \varepsilon$, i.e.

$|f(x) - 7| < \varepsilon$ and hence $\lim_{x \to 4} f(x)$ is indeed 7. $\square$

$\varepsilon - \delta$ Definition of the Limit:

$\lim_{x \to c} f(x) = L$ if given $\varepsilon > 0$, there exists a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.

$f(x) = -5x + 3$; find $\lim_{x \to 1} f(x)$ & find a δ.

$L = -5(1) + 3 = -2$, $c = 1$

$|f(x) - L| = |-5x + 3 - (-2)| = |-5x + 5| =$

$= |-5(x-1)| = \frac{5|x-1|}{5} < \frac{\varepsilon}{5} = \delta$

Prove that the limit is L using the $\varepsilon - \delta$ definition of the limit.

28. $\lim_{x \to -3} (2x + 5)$

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1.2 # 39, 40, 41