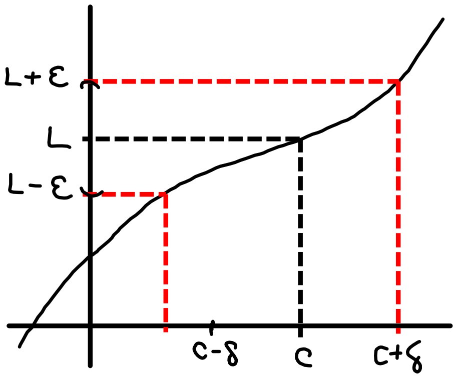
Differential Calculus 1.2-1.3 - Epsilon-Delta & Exaluating Limits Analytically 10 Nov. 2017

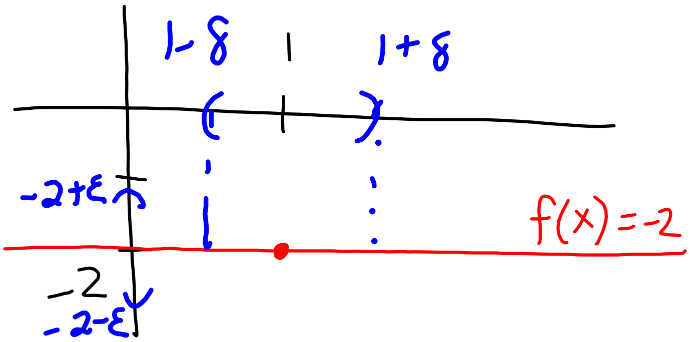
$\varepsilon - \delta$ Definition of the Limit:

$\lim_{x\to c} f(x) = L$ if given $\varepsilon > 0$, there exists a $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.

$f(x) = -2$

$\lim_{x\to 1} (-2) = -2$





$|f(x) - L| = |-2-(-2)| = 0 < \varepsilon$

what δ, will $|x-c| < \delta$ imply this

Let $\delta = \varepsilon$

Prove that the limit is L using the $\varepsilon - \delta$ definition of the limit.

28. $\lim_{x\to -3} (2x + 5)$

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Find δ for $\varepsilon = 0.01$

24. $\lim_{x\to 4}\left(4 - \frac{x}{2}\right) = 4 - \frac{4}{2} = 4 - 2 = 2$

$L = 2, \quad c = 4$

$f(x) = 4 - \frac{x}{2}$

$|f(x) - L| = \left|4 - \frac{x}{2} - 2\right| = \left|-\frac{x}{2} + 2\right| = \left|-\frac{1}{2}(x-4)\right|$

$= \frac{\frac{1}{2}|x-4|}{1/2} < \frac{0.01}{1/2}$

$\boxed{\delta = 0.02}$

Find δ for $\varepsilon = 0.01$

26. $\lim_{x\to 5}(x^2 + 4) = 5^2 + 4 = 29$

$|f(x) - L| = |x^2 + 4 - 29| = |x^2 - 25| =$

$= |(x+5)(x-5)| < \frac{11|x-5|}{11} < \frac{\varepsilon}{11}$

[If $x \to 5$, then $x < 6 \Rightarrow x + 5 < 6 + 5 = 11$]

Take $\delta = \varepsilon/11$

1.3 Evaluating Limits Analytically

If $\lim_{x \to c} f(x) = f(c)$,

we say that $f(x)$ is

continuous at c.

Evaluating Limits Analytically

Basic Limits

Let $b, c \in \mathbb{R}$, $n > 0$ *an integer*, $f, g -$ *functions*, $\lim_{x\to c} f(x) = L$, $\lim_{x\to c} g(x) = K$

1. Constant	$\lim_{x\to c} b = b$	
2. Identity	$\lim_{x\to c} x = c$	$\lim_{x\to c} [f(x) \pm g(x)] = \lim_{x\to c} f(x) \pm \lim_{x\to c} g(x)$
3. Polynomial	$\lim_{x\to c} x^n = c^n$	
4. Scalar Multiple	$\lim_{x\to c}[bf(x)] = bL$	$\lim_{x\to c} [f(x) g(x)] = \left[\lim_{x\to c} f(x)\right] \cdot \left[\lim_{x\to c} g(x)\right]$
5. Sum or Difference	$\lim_{x\to c}[f(x) \pm g(x)] = L \pm K$	
6. Product	$\lim_{x\to c}[f(x)g(x)] = LK$	$\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}$, $\lim_{x\to c} g(x) \neq 0$
7. Quotient	$\lim_{x\to c}\left[\frac{f(x)}{g(x)}\right] = \frac{L}{K}$, $K \neq 0$	
8. Power	$\lim_{x\to c}[f(x)]^n = L^n$	$\lim_{x\to c} [f(x)]^n = \left[\lim_{x\to c} f(x)\right]^n$

Note: If substitution yields $\frac{0}{0}$, an indeterminate form, the expression must be rewritten in order to evaluate the limit.

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$$\lim_{x\to c} a = a$$

$$\lim_{x\to c} x = c$$

$$\lim_{x\to c} x^n = c^n$$

$$\lim_{x\to 5} (-3) = -3$$

$$\lim_{x\to -\pi} x = -\pi$$

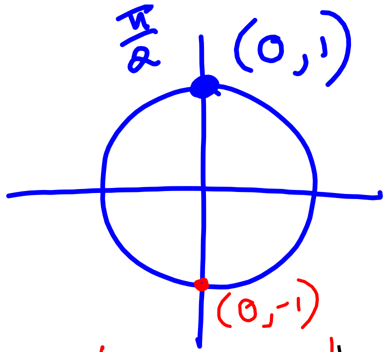
$$\lim_{x\to -1} x^5 = (-1)^5 = -1$$

1.3

12. $\lim_{x\to 1} \left(3x^3 - 2x^2 + 4\right) = 3\cdot 1^3 - 2\cdot 1^2 + 4 = \boxed{5}$

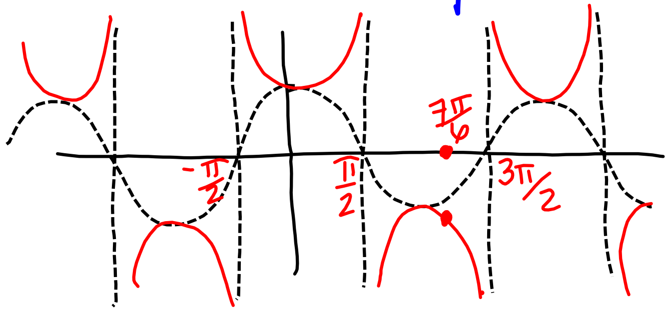
18. $\lim_{x\to 3} \dfrac{\sqrt{x+1}}{x-4} = \dfrac{\sqrt{3+1}}{3-4} = \boxed{-2}$

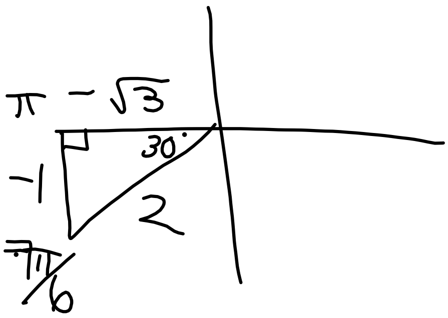
30. $\lim_{x\to 1} \sin \frac{\pi x}{2} = \sin \frac{\pi}{2} = 1$



36. $\lim_{x\to 7} \sec\left(\frac{\pi x}{6}\right)$

$= \sec \frac{7\pi}{6} = \boxed{-\frac{2}{\sqrt{3}}}$





1.2 #33