$$\lim_{x\to 2}(3x+2)=8 \qquad \varepsilon = 0.01$$

$$|f(x)-L| = |3x+2-8| = |3x-6| = \frac{3|x-2|}{3} < \frac{0.01}{3}$$

$$\text{Take } \delta = \frac{0.01}{3} = \boxed{0.00\overline{3}}$$

$$\frac{1}{3} = 0.\overline{3}$$

1.3 Evaluating Limits Analytically

If $\lim_{x\to c} f(x) = f(c)$,

we say that $f(x)$ is

continuous at c.

p56
1.2 # 39, 40, 41

1.2 # 33

Evaluating Limits Analytically

Basic Limits

Let $b, c \in \mathbb{R}$, $n > 0$ *an integer*, f, g – *functions*, $\lim_{x\to c} f(x) = L$, $\lim_{x\to c} g(x) = K$

1. Constant $\lim_{x\to c} b = b$
2. Identity $\lim_{x\to c} x = c$
3. Polynomial $\lim_{x\to c} x^n = c^n$
4. Scalar Multiple $\lim_{x\to c}[bf(x)] = bL$
5. Sum or Difference $\lim_{x\to c}[f(x) \pm g(x)] = L \pm K$ $\quad \lim_{x\to c}[f(x) \pm g(x)] = \lim_{x\to c} f(x) \pm \lim_{x\to c} g(x)$
6. Product $\lim_{x\to c}[f(x)g(x)] = LK$ $\quad \lim_{x\to c}[f(x)g(x)] = \left[\lim_{x\to c} f(x)\right]\cdot\left[\lim_{x\to c} g(x)\right]$
7. Quotient $\lim_{x\to c}\left[\frac{f(x)}{g(x)}\right] = \frac{L}{K}$, $K \neq 0$ $\quad \lim_{x\to c}\frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}$, $\lim_{x\to c} g(x) \neq 0$
8. Power $\lim_{x\to c}[f(x)]^n = L^n$ $\quad \lim_{x\to c}[f(x)]^n = \left[\lim_{x\to c} f(x)\right]^n$

Note: If substitution yields $\frac{0}{0}$, an indeterminate form, the expression must be rewritten in order to evaluate the limit.

38. $\lim_{x\to c} f(x) = \frac{3}{2}$; $\lim_{x\to c} g(x) = \frac{1}{2}$

(a) $\lim_{x\to c}[4f(x)] = 4\left(\frac{3}{2}\right) = \boxed{6}$

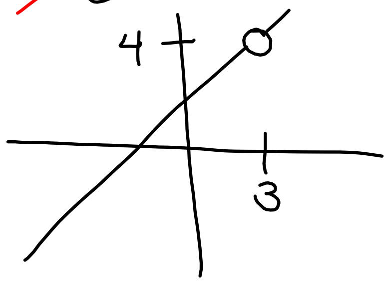
(b) $\lim_{x\to c}[f(x)+g(x)] = \frac{3}{2} + \frac{1}{2} = \boxed{2}$

(c) $\lim_{x\to c}[f(x)g(x)] = \frac{3}{2}\cdot\frac{1}{2} = \boxed{\frac{3}{4}}$

(d) $\lim_{x\to c}\frac{f(x)}{g(x)} = \frac{3/2}{1/2} = \frac{3}{2}\cdot\frac{2}{1} = \boxed{3}$

$$\lim_{x\to 3} \frac{x^2-2x-3}{x-3} = \lim_{x\to 3} \frac{\cancel{(x-3)}(x+1)}{\cancel{x-3}} = 3+1 = \boxed{4}$$

$$\frac{\cancel{(x-3)}(x+1)}{\cancel{x-3}} = x+1,\ x \neq 3$$



$$\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2} =$$

$$= \lim_{x\to 4} \frac{\cancel{(x-4)}(\sqrt{x}+2)}{\cancel{x-4}} = \sqrt{4}+2 = \boxed{4}$$

Given $f(x) = 2x^2 + 3x + 1$

$f(x+h) \neq f(x) + h$

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{2(x+h)^2 + 3(x+h) + 1 - (2x^2+3x+1)}{h}$$

$$= \lim_{h\to 0} \frac{2(x^2+2xh+h^2) + 3x + 3h + 1 - 2x^2 - 3x - 1}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 + \cancel{3x} + 3h + \cancel{1} - \cancel{2x^2} - \cancel{3x} - \cancel{1}}{h}$$

$$= \lim_{h\to 0} \frac{4xh + 2h^2 + 3h}{h} = \lim_{h\to 0} \frac{\cancel{h}(4x+2h+3)}{\cancel{h}}$$

$$= 4x + 2(0) + 3$$

$$= \boxed{4x+3}$$

The Binomial Theorem

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

$(a+b)^0 = 1$

$(a+b)^1 = a+b$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

Pascal's $\triangle$

$$\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}$$

$f(x) = x^3$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^3}+3x^2h+3xh^2+h^3-\cancel{x^3}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(3x^2+3xh+h^2)}{\cancel{h}} = \boxed{3x^2}$$

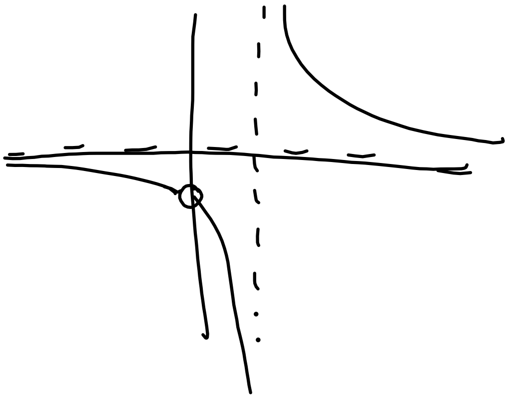
1.3 Evaluating Limits Analytically

42. $h(x) = \dfrac{x^2 - 3x}{x}$

(a) $\lim\limits_{x\to -2} h(x) = \dfrac{(-2)^2-3(-2)}{-2} = \dfrac{4+6}{-2} = \boxed{-5}$

(b) $\lim\limits_{x\to 0} h(x) = \lim\limits_{x\to 0} \dfrac{\cancel{x}(x-3)}{\cancel{x}} = \boxed{-3}$

44. $\lim\limits_{x\to 1} \dfrac{x}{x^2 - x} = \lim\limits_{x\to 1} \dfrac{\cancel{x}}{\cancel{x}(x-1)} = \lim\limits_{x\to 1} \dfrac{1}{x-1}$ $\boxed{\text{DNE}}$



$\lim\limits_{x\to 1^-} \dfrac{1}{x-1} = -\infty$

$\lim\limits_{x\to 1^+} \dfrac{1}{x-1} = \infty$

$\dfrac{1}{x} \to \dfrac{1}{10}, \dfrac{1}{100}, \dfrac{1}{1000}, \dfrac{1}{1000000}, \ldots$

$\to 0$ as $x \to \infty$

$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$

48. $\lim\limits_{x\to -1} \dfrac{x^3 + 1^3}{x+1} = \lim\limits_{x\to -1} \dfrac{\cancel{(x+1)}(x^2 - x + 1)}{\cancel{x+1}}$

$= (-1)^2 - (-1) + 1 = \boxed{3}$

54. $\lim\limits_{x\to 0} \dfrac{\sqrt{2+x}-\sqrt{2}}{x} \cdot \dfrac{\sqrt{2+x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{2}}$

$= \lim\limits_{x\to 0} \dfrac{\cancel{2}+\cancel{x}^{\,1} \cancel{-2}}{\cancel{x}(\sqrt{2+x}+\sqrt{2})} = \boxed{\dfrac{1}{2\sqrt{2}}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \boxed{\dfrac{\sqrt{2}}{4}}$

58. $\lim\limits_{x\to 0} \dfrac{\dfrac{1}{x+4}-\dfrac{1}{4}}{x} = \lim\limits_{x\to 0} \dfrac{\dfrac{1}{x+4}\cdot\dfrac{4}{4} - \dfrac{1}{4}\cdot\dfrac{x+4}{x+4}}{\left(\dfrac{x}{1}\right)}$

$= \lim\limits_{x\to 0} \dfrac{4-(x+4)}{4(x+4)} \cdot \dfrac{1}{x}$

$= \lim\limits_{x\to 0} \dfrac{-\cancel{x}}{4(x+4)} \cdot \dfrac{1}{\cancel{x}} = \lim\limits_{x\to 0} \dfrac{-1}{4(x+4)} = \boxed{\dfrac{-1}{16}}$

66. $\lim_{x\to 2} \frac{x^5 - 32}{x - 2} = \lim_{x\to 2} \frac{\cancel{(x-2)}(x^4+2x^3+4x^2+8x+16)}{\cancel{x-2}}$

$$\begin{array}{r|rrrrrr} 2 & 1 & 0 & 0 & 0 & 0 & -32 \\ & \downarrow & 2 & 4 & 8 & 16 & 32 \\ \hline & 1 & 2 & 4 & 8 & 16 & 0 \end{array}$$

$= 2^4 + 2(2)^3 + 4(2)^2 + 8(2) + 16 = \boxed{80}$