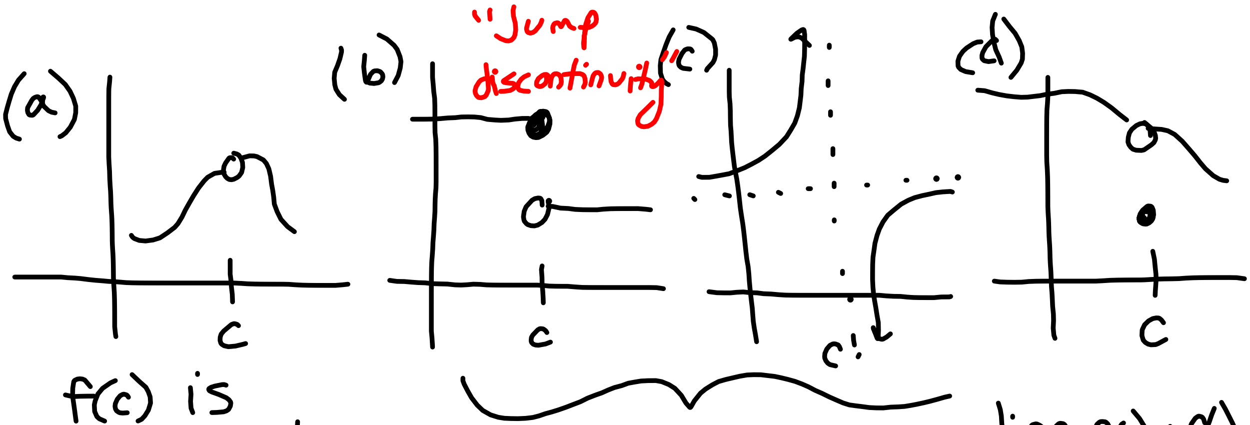
1.4 Continuity and One-Sided Limits



f(c) is undefined

$\lim_{x \to c} f(x)$ does not exist

$\lim_{x \to c} f(x) \neq f(c)$

These are all discontinuities
(a) and (d) are removable
(b) and (c) are nonremovable

Continuity at a point

A function f is continuous at c if the following 3 conditions are met:

1. $f(c)$ is defined
2. Limit of $f(x)$ exists when x approaches c
3. Limit of $f(x)$ when x approaches c is equal to $f(c)$

f(x) is continuous at c if
$$\lim_{x \to c} f(x) = f(c)$$

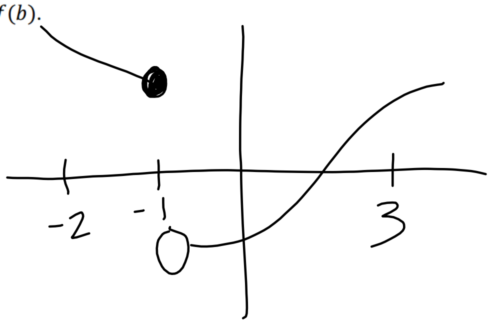
Continuity on an open interval

A function is continuous on an open interval if it is continuous at each point in the interval. A function that is continuous on the entire real line $(-\infty, \infty)$ is everywhere continuous.

Continuity on a closed interval

A function f is continuous on the closed interval $[a, b]$ if it is continuous on the open interval I (a, b) and $\lim_{x \to a^+} f(x) = f(a)$ and $\lim_{x \to b^-} f(x) = f(b)$.

$[a \quad , \quad b]$



f is cts on:
$[-2, -1]$
$(-1, 3]$

10. $\lim\limits_{x \to 4^-} \dfrac{\sqrt{x} - 2}{x - 4}$

12. $\lim\limits_{x \to 2^+} \dfrac{|x - 2|}{x - 2}$

1.4

Discuss the [dis]continuity of the function.

$$f(x) = \frac{(x+4)(x-2)}{(x-2)(x+1)}$$

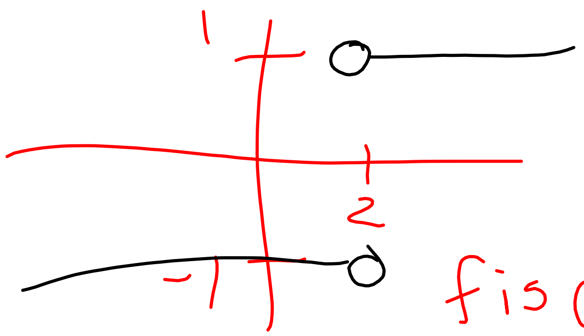
removable discontinuity @ $x=2$

non-removable discontinuity @ $x=-1$ vertical asymptote

f is continuous on:

$(-\infty,-1)\cup(-1,2)\cup(2,\infty)$

$$f(x) = \frac{|x-2|}{x-2} = \begin{cases} 1, & x>2 \\ -1, & x<2 \end{cases}$$



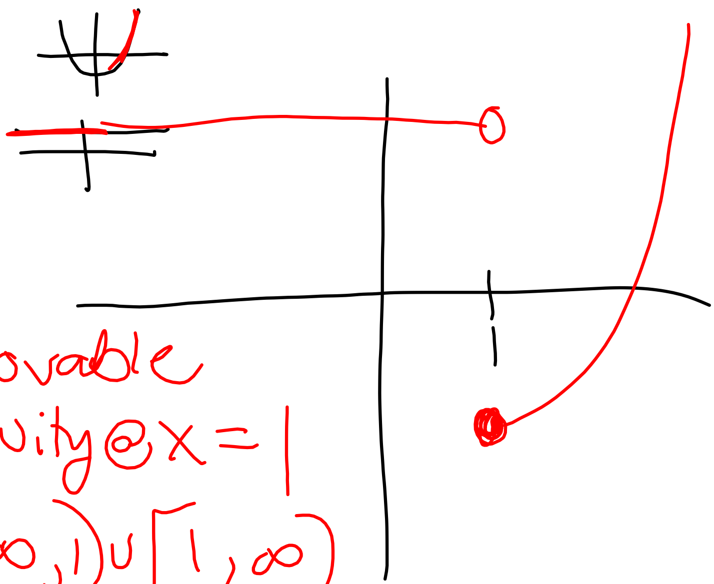
f has a nonremovable (jump) discontinuity @ $x=2$

f is cts on: $(-\infty,2)\cup(2,\infty)$

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$\frac{|x-2|}{x-2} = \begin{cases} \frac{x-2}{x-2} = 1, & x-2 > 0 \\ & x > 2 \\ \frac{-(x-2)}{x-2} = -1, & x-2 < 0 \\ & x < 2 \end{cases}$$

$$f(x) = \begin{cases} x^2-2, & x \geq 1 \\ 5, & x < 1 \end{cases}$$

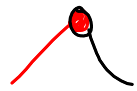


$1^2-2 = -1 \neq 5$

f has a non-renovable (jump) discontinuity @ $x=1$

f is cts on: $(-\infty, 1) \cup [1, \infty)$

$$f(x) = \begin{cases} x+6, & x \le -2 \\ x^2, & -2 < x \le 3 \\ 8 & x > 3 \end{cases}$$



$-2+6=4$

$(-2)^2=4$ $\quad 3^2=9 \neq 8$

f has a non-removable jump discontinuity @ $x=3$

f is cts on: $(-\infty, 3] \cup (3, \infty)$

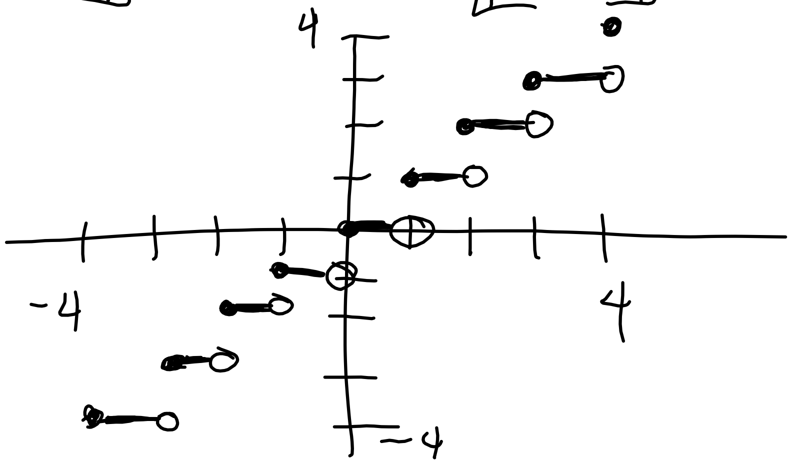
$$f(x) = \begin{cases} \dfrac{|x-3|}{3-x}, & |x-3| > 5 \\ x^2-3, & -2 \le x \le 8 \end{cases}$$

The Greatest Integer Function

$[\![x]\!]$ = the greatest integer less than or equal to x

$[\![-2]\!] = -2$ $\qquad$ $[\![\pi]\!] = 3$

$[\![7{,}000]\!] = 7{,}000$ $\qquad$ $[\![-\sqrt{2}]\!] = -2$

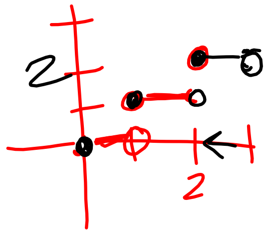


$\lfloor x \rfloor$

$\lceil x \rceil$

22. $\lim\limits_{x\to 2^+} 2x - [\![x]\!]$

$= \lim\limits_{x\to 2^+} 2x - \lim\limits_{x\to 2^+} [\![x]\!]$



$= 4 - 2 = \boxed{2}$

x	2.1	2.001
$[\![x]\!]$	2	2

24. $\lim_{x \to 1} \left(1 - \left[\!\left[-\frac{x}{2} \right]\!\right]\right)$

x	0.99	1.001
$\left[\!\left[-\frac{x}{2} \right]\!\right]$	$\left[\!\left[\frac{-0.99}{2} \right]\!\right]$	$\left[\!\left[\frac{-1.0}{2} \right]\!\right]$

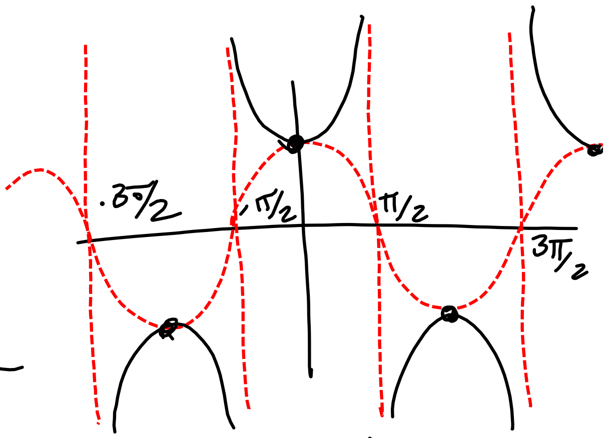
$= \lim_{x \to 1} 1 - \lim_{x \to 1} \left[\!\left[-\frac{x}{2} \right]\!\right]$

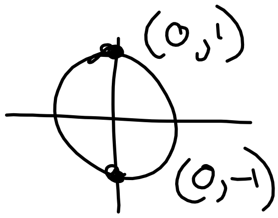
$= 1 - (-1) = \boxed{2}$

20. $\lim_{x \to \frac{\pi}{2}} \sec x$

does not exist

$\lim_{x \to \frac{\pi}{2}^-} \sec x = \infty$





52. $f(x) = \tan\frac{\pi x}{2}$

discuss the (dis)continuity

tanx has period π

$y = a \cdot f(bx+c) + d$

period: $\frac{\text{original per}}{|b|}$

$= \frac{\pi}{\pi/2} = 2$