$$\lim_{x\to -2} h(g(x)) = \lim_{g(x)\to 0} h(g(x)) = \begin{cases} 1 \\ -1 \end{cases} \text{DNE}$$

$$\lim_{x\to -2} g(x) = 0$$

$$f(x) = 2x^2 - 3x$$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{2(x+h)^2 - 3(x+h) - (2x^2-3x)}{h}$$

$$= \lim_{h\to 0} \frac{\overset{\cancel{2x^2}+4xh+2h^2}{2(x^2+2xh+h^2)} - \cancel{3x} - 3h - \cancel{2x^2} + \cancel{3x}}{h}$$

$$= \lim_{h\to 0} \frac{4xh + 2h^2 - 3h}{h} = \lim_{h\to 0} \frac{\cancel{h}(4x+2h-3)}{\cancel{h}} = \boxed{4x-3}$$

$$\lim_{x\to 0}\left(5x^2 \sin\frac{1}{x}\right)$$

$$-1 \le \sin\frac{1}{x} \le 1$$

$$-5x^2 \le 5x^2\sin\frac{1}{x} \le 5x^2$$

$$\lim_{x\to 0}\left(-5x^2\right) \le \lim_{x\to 0} 5x^2\sin\frac{1}{x} \le \lim_{x\to 0} 5x^2$$

$$0 \le \qquad \le 0$$

$$\lim_{x\to 0} 5x^2\sin\frac{1}{x} = \boxed{0}$$

$$f(x) = \frac{x^2-7x+10}{x^2-3x+2}$$

$$= \frac{(x-5)\cancel{(x-2)}}{(x-1)\cancel{(x-2)}}$$

f has a removable (hole) discontinuity @ $x=2$ &

f has a non-removable (vertical asymptote) discontinuity @ $x=1$

f is continuous on $(-\infty,1)\cup(1,2)\cup(2,\infty)$

62. $f(x)=\frac{1}{\sqrt{x}}$, $g(x)=x-1$

domain: $(0,\infty)$
continuous on its domain

domain: $(-\infty,\infty)$
continuous on its domain

Discuss the continuity of $f(g(x))$.

$f(g(x))=\frac{1}{\sqrt{x-1}}$

domain:
$\{x \mid x-1>0\}$
$x>1$

continuous on $(1,\infty)$

64. $f(x)=\sin x$; $g(x)=x^2$

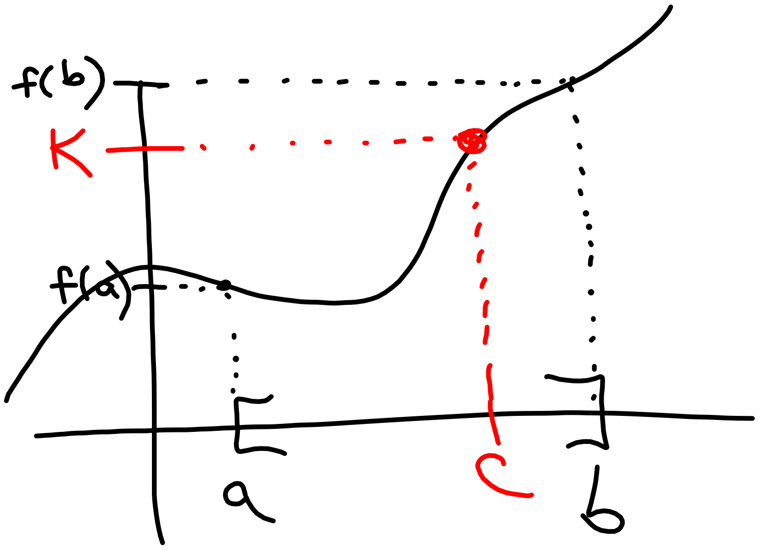
discuss the continuity of $f(g(x))$

$f(g(x)) = \sin(x^2)$

continuous on $(-\infty,\infty)$

Intermediate Value Theorem

If f is continuous on the closed interval $[a,b]$ and k is any number between $f(a)$ and $f(b)$, then there is at least one number c in $[a,b]$ such that $f(c)=k$.



Does the IVT guarantee a zero in the given interval?

76. $f(x)=x^3+3x-2$, $[0,1]$

$f(0)=-2<0$

$f(1)=1+3-2=2>0$

IVT guarantees a zero

$x^3+3x-2=0$

solve$(x^3+3x-2=0,x)$

$x=0.596072$

84. $f(x) = x^2 - 6x + 8$; $[0, 3]$; $f(c) = 0$

$f(0) = 8 > 0$

$f(3) = 3^2 - 6 \cdot 3 + 8 = -1 < 0$ } yes

$x^2 - 6x + 8 = 0$

$(x-4)(x-2) = 0$

$\cancel{x = 4}$ $\cancel{\in [0,3]}$, $x = 2$

86. $f(x) = \dfrac{x^2 + x}{x - 1}$, $\left[\frac{5}{2}, 4\right]$, $f(c) = 6$

$f(5/2) = \dfrac{\left(\frac{5}{2}\right)^2 + \frac{5}{2}}{\frac{5}{2} - 1} = \dfrac{\frac{25}{4} + \frac{10}{4}}{\frac{5}{2} - \frac{2}{2}} = \dfrac{35}{\cancel{4}_2} \cdot \dfrac{\cancel{2}}{3} = \dfrac{35}{6} < 6$

$f(4) = \dfrac{4^2 + 4}{4 - 1} = \dfrac{16 + 4}{3} = \dfrac{20}{3} > 6$

IVT guarantees a $c \in \left[\frac{5}{2}, 4\right]$ st. $f(c) = 6$

$\dfrac{x^2 + x}{x - 1} = 6 \Rightarrow x^2 + x = 6(x - 1)$

$x^2 + x = 6x - 6$

$x^2 - 5x + 6 = 0$

$(x-3)(x-2) = 0$

$\cancel{x = 2}$, $x = 3$

1.5

Infinite Limits

$$\lim_{x \to c} f(x) = \pm\infty$$

means the function increases or decreases without bound; i.e. the graph of the function approaches a vertical asymptote

Finding Vertical Asymptotes

x-values at which a function is undefined result in either holes in the graph or vertical asymptotes. Holes result when a function can be rewritten so that the factor which yields the discontinuity cancels. Factors that can't cancel yield vertical asymptotes.

Examples:

$f(x) = \frac{1}{x(x+3)}$ has vertical asymptotes at $x = 0$ and $x = 3$

$f(x) = \frac{(x+2)(x+3)}{x(x+3)}$ has a vertical asymptote at $x = 0$ and a hole at $x = -3$

Rules involving infinite limits

Let $\lim_{x \to c} f(x) = \infty \ and \ \lim_{x \to c} g(x) = L$

1. $\lim_{x \to c} [f(x) \pm g(x)] = \infty$

2. $\lim_{x \to c} [f(x)g(x)] = \begin{cases} \infty, & L > 0 \\ -\infty, & L < 0 \end{cases}$

3. $\lim_{x \to c} \frac{g(x)}{f(x)} = 0$

Find the vertical asymptotes (if any).

14. $f(x) = \dfrac{-4x}{x^2+4}$

no vertical asymptotes

24. $h(x) = \dfrac{x^2-4}{\underbrace{x^3+2x^2}_{x^2(x+2)}+\underbrace{x+2}_{+1(x+2)}} = \dfrac{(x+2)(x-2)}{(x+2)(x^2+1)}$

no vertical asymptotes

28. $g(\theta) = \dfrac{\tan\theta}{\theta} = \dfrac{\sin\theta}{\theta}\cdot\dfrac{1}{\cos\theta}$

V.A's @ all odd multiples of $\pi/2$