Rules involving infinite limits

Let $\lim_{x\to c} f(x) = \infty$ *and* $\lim_{x\to c} g(x) = L$

1. $\lim_{x\to c}[f(x) \pm g(x)] = \infty$

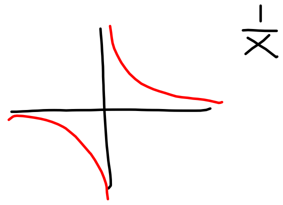
2. $\lim_{x\to c}[f(x)g(x)] = \begin{cases} \infty, & L > 0 \\ -\infty, & L < 0 \end{cases}$

3. $\lim_{x\to c} \dfrac{g(x)}{f(x)} = 0$

42. $\lim_{x\to 0^-}\left(x^2 - \frac{1}{x}\right) = \lim_{x\to 0^-} x^2 - \lim_{x\to 0^-}\frac{1}{x}$

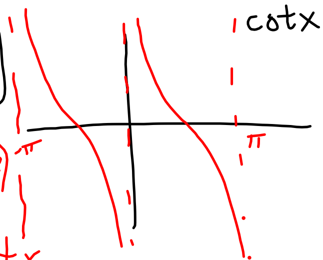
$= 0 - (-\infty)$

$= \boxed{\infty}$



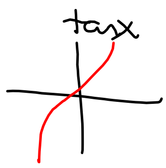
46. $\lim_{x\to 0}\dfrac{x+2}{\cot x} \to \dfrac{2}{\pm\infty} = \boxed{0}$

$\dfrac{\lim_{x\to 0^-}(x+2)}{\lim_{x\to 0^-}\cot x} = \dfrac{2}{-\infty} = 0 = \dfrac{2}{\infty} = \dfrac{\lim_{x\to 0^+}(x+2)}{\lim_{x\to 0^+}\cot x}$



$= \lim_{x\to 0}(x+2)(\tan x)$

$= 2\cdot 0 = \boxed{0}$

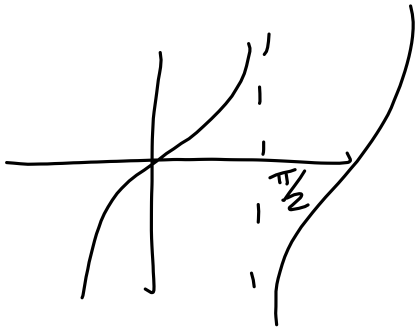


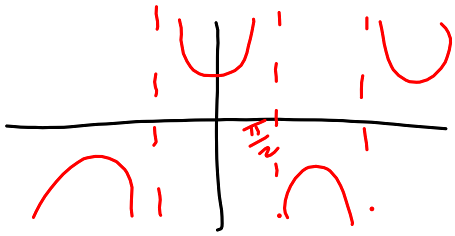
48. $\lim_{x \to \frac{1}{2}} x^2 \tan \pi x$

$= \left(\lim_{x \to \frac{1}{2}} x^2\right)\left(\lim_{x \to \frac{1}{2}} \tan \pi x\right)$

$= \frac{1}{4} \cdot \lim_{x \to \frac{\pi}{2}} \tan x$

$= \frac{1}{4} \cdot \boxed{\text{DNE}}$





52. $\lim_{x \to 3^+} \sec \frac{\pi x}{6} = \lim_{x \to \frac{\pi}{2}^+} \sec x = \boxed{-\infty}$

2.1 The Derivative & The Tangent Line Problem

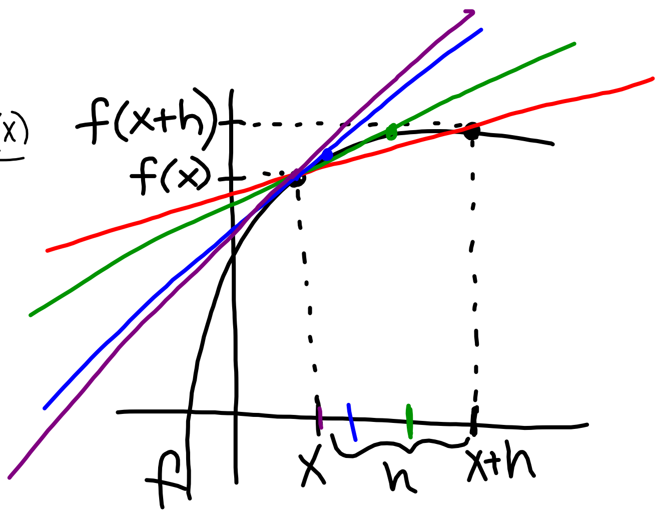
secant line crosses through a function at two points

slope of the secant line:

$$\frac{f(x+h)-f(x)}{x+h-x} = \frac{f(x+h)-f(x)}{h}$$

what happens as $h \to 0$?

$$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$



As $h \to 0$, the secant line approximates the tangent line, and the limit is the slope of the tangent line and we call it **the derivative of f at x**.

[redacted]

$\boldsymbol{f'(x)}$	"f prime of x"
$\boldsymbol{\frac{dy}{dx}} = \frac{d}{dx}(y)$	"derivative of y with respect to x"
$\boldsymbol{y'}$	"y prime"
$\boldsymbol{\frac{d}{dx}[f(x)]}$	"the derivative with respect to x of $f(x)$"
$\boldsymbol{D_x[y]}$	"the partial derivative with respect to x of y"

The Derivative

The slope of the tangent line to the graph of f at the point $(c, f(c))$ is given by:

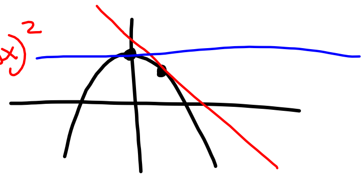
$$m = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x} = f'(c)$$

The derivative of f at x is given by

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

8. $g(x) = 5 - x^2$

$g(c+\Delta x) = 5-(c+\Delta x)^2$



find slope of tangent line at the points $(\overset{c}{2},\overset{g(c)}{1})$ & $(0,5)$

$$g'(c) = \lim_{\Delta x \to 0} \frac{g(c+\Delta x)-g(c)}{\Delta x}$$

$$g'(2) = \lim_{\Delta x \to 0} \frac{5-(2+\Delta x)^2 - 1}{\Delta x} = \lim_{\Delta x \to 0} \frac{5-(4+4\Delta x+(\Delta x)^2)-1}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{5}-\cancel{4}-4\Delta x-(\Delta x)^2\cancel{-1}}{\Delta x} = \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(-4-\Delta x)}{\cancel{\Delta x}}$$

$= \boxed{-4}$ ← slope @ $(2,1)$

to find eq:
$y-y_1 = m(x-x_1)$
...
$y = mx+b$

$$g'(c) = \lim_{\Delta x \to 0} \frac{g(c+\Delta x)-g(c)}{\Delta x}$$

$g(x) = 5-x^2$
$(c, g(c)) = (0,5)$

$$g'(0) = \lim_{\Delta x \to 0} \frac{5-(0+\Delta x)^2-5}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-(\Delta x)^2}{\Delta x} = \lim_{\Delta x \to 0} (-\Delta x) = \boxed{0}$$

← slope of tangent line to $g(x)$ @ $(0,5)$

20. $f(x) = x^3 + x^2$

find the derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^3+(x+h)^2-(x^3+x^2)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^3}+3x^2h+3xh^2+h^3+\cancel{x^2}+2xh+h^2-\cancel{x^3}-\cancel{x^2}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(3x^2+3xh+h^2+2x+h)}{\cancel{h}} = \boxed{3x^2+2x}$$

Find the equation of the tangent line to $f(x)=x^3-x$ at the point $(2,6)$.

x_1, y_1

$m = f'(2) = 3(2)^2 - 1 = 11$

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^3-(x+h)-(x^3-x)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^3}+3x^2h+3xh^2-\cancel{x}-h-\cancel{x^3}+\cancel{x}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(3x^2+3xh-1)}{\cancel{h}} = 3x^2-1 = f'(x)$$

$y - y_1 = m(x - x_1)$

$y - 6 = 11(x-2)$

$y = 11x - 22 + 6$

$\boxed{y = 11x - 16}$