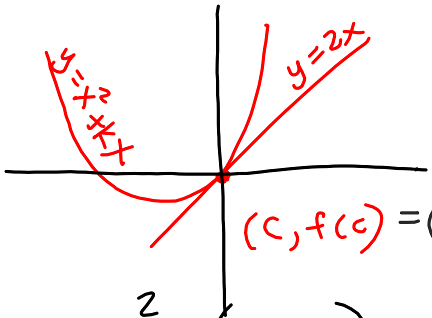
Find k s.t. $y=2x$ is tangent to $f(x)=x^2+kx$.

$y = mx + b$, $m = 2$, $b = 0$

$$\begin{cases} f'(x) = 2x+k \\ f'(c) = 2 \end{cases}$$



$(c, f(c)) = (0,0)$

$$\boxed{2x+k=2 \qquad 2x = x^2+kx}$$

$$\frac{2x}{2} = \frac{2-k}{2}$$

$$x = 1 - \frac{k}{2}$$

$$k = 2-2x$$

$$= 2-2(0)$$

$$\boxed{k=2}$$

$$2x = x^2 + (2-2x)x$$

$$2x = x^2 + 2x - 2x^2$$

$$0 = -x^2 \Rightarrow x = 0$$

$$f(x) = (x+4)^{2/3}, \quad c = -4 \qquad f'(c) = \lim_{x \to c} \frac{f(x)-f(c)}{x-c}$$

$$f'(c) = \lim_{x \to -4} \frac{(x+4)^{2/3} - (-4+4)^{2/3}}{x-(-4)}$$

$$= \lim_{x \to -4} \frac{(x+4)^{2/3}}{(x+4)^{3/3}} = \lim_{x \to -4} \frac{1}{(x+4)^{1/3}} = \lim_{x \to -4} \frac{1}{\sqrt[3]{x+4}} \to \pm\infty$$

$\Rightarrow$ Vertical tangent line; $\boxed{f'(-4) \text{ DOES NOT EXIST}}$

The Derivative

The slope of the tangent line to the graph of f at the point $(c, f(c))$ is given by:

$$m = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(c + \Delta x) - f(c)}{\Delta x}$$

The derivative of f at x is given by

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

2.2 Basic Differentiation Rules

1. The derivative of a constant function is zero, i.e., for $c \in \mathbb{R}$, $\frac{d}{dx}[c] = 0$

2. Power Rule for $n \epsilon \mathbb{Q}$, $\frac{d}{dx}[x^n] = nx^{n-1}$

3. Constant Multiple Rule $\in \mathbb{R}$, $\frac{d}{dx}[cf(x)] = cf'(x)$

4. Sum & Difference Rules $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

Derivatives of Trig Functions

1. $\frac{d}{dx}[\sin x] = \cos x$
2. $\frac{d}{dx}[\cos x] = -\sin x$
3. $\frac{d}{dx}[\tan x] = \sec^2 x$
4. $\frac{d}{dx}[\cot x] = -\csc^2 x$
5. $\frac{d}{dx}[\sec x] = \sec x \tan x$
6. $\frac{d}{dx}[\csc x] = -\csc x \cot x$

Proof that (sin x)'=cos x

$$\frac{d}{dx}(\sin x) = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h} =$$

$$= \lim_{h \to 0} \frac{\sin x \cos h + \sin h \cos x - \sin x}{h}$$

$$= \lim_{h \to 0} \left[-\frac{\sin x(1 - \cos h)}{h} + \frac{\sin h \cos x}{h} \right]$$

$$= (-\sin x)(0) + (1)\cos x = \boxed{\cos x}$$

2.2

22. $y = 5 + \sin x$

$y' = \boxed{\cos x}$

$\frac{5}{8x^3} = \frac{5}{8} \cdot \frac{1}{x^3} = \frac{5}{8}x^{-3}$

24. $y = \frac{5}{(2x)^3} + 2\cos x = \frac{5}{8}x^{-3} + 2\cos x$

$y' = \boxed{\frac{-15}{8}x^{-4} - 2\sin x} = -\frac{15}{8x^4} - 2\sin x$

44. $h(x) = \frac{2x^3 - 3x + 1}{x} = \frac{2x^3}{x} - \frac{3x}{x} + \frac{1}{x} = 2x^2 - 3 + x^{-1}$

$h'(x) = \boxed{4x - x^{-2}} = 4x - \frac{1}{x^2} = \frac{4x^3 - 1}{x^2}$

46. $y = 3x(6x - 5x^2) = 18x^2 - 15x^3$

$y' = \boxed{36x - 45x^2}$

52. $f(x) = \frac{2}{\sqrt[3]{x}} + 3\cos x = 2x^{-1/3} + 3\cos x$

$f'(x) = \boxed{-\frac{2}{3}x^{-4/3} - 3\sin x}$

$= -\frac{2}{3\sqrt[3]{x^4}} - 3\sin x$

2.2 cont.

$s(t)$ = position

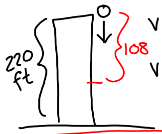
$v(t) = s'(t)$ = velocity (rate of change of position)

$a(t) = v'(t) = s''(t)$ = acceleration (rate of change of velocity)

average velocity: $\frac{\Delta s}{\Delta t}$ (slope of secant)

instantaneous velocity $= s'(t)$ (slope of tangent)

92. initial velocity $V_0 = -22$ ft/s



$V(3) = ?$

$V(t) = ?$ after falling 108 ft

$g = -9.8\ m/s^2$
$= -32\ ft/s^2$

$s(t) = \frac{1}{2}at^2 + v_0 t + S_0$

$s(t) = \frac{1}{2}(-32)t^2 + (-22)t + 220$

$s(t) = -16t^2 - 22t + 220$
$V(t) = s'(t) = -32t - 22$ $(= at + V_0)$

$V(3) = -32(3) - 22 = \boxed{-118\ ft/s}$

$V(t) = ?$ when $s(t) = 220 - 108 = 112$

$112 = -16t^2 - 22t + 220$

$16t^2 + 22t - 108 = 0$

$8t^2 + 11t - 54 = 0$

$\underbrace{2 \cdot 2 \cdot 2 \cdot 2}_{16} \cdot \underbrace{3 \cdot 3 \cdot 3}_{-27}$

$8t^2 - 16t + 27t - 54 = 0$

$8t(t-2) + 27(t-2) = 0$

$(8t + 27)(t-2) = 0$

$t = \frac{-27}{8}$, $t = 2 \rightarrow V(2) = -32(2) - 22$
$= \boxed{-86\ ft/s}$