The volume of a sphere is given by

Find the rate of change of volume of a sphere with respect to radius when the radius is 2 cm.

$$V(r) = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dr} = V'(r) = \frac{4}{3}\pi(3r^2) = 4\pi r^2 = \text{surface area of sphere}$$

$$V'(2) = 4\pi(2)^2 = \boxed{16\pi \text{ cm}^2}$$

Find the average rate of change of volume as the radius changes from 1 cm to 3 cm.

$$\frac{\Delta V}{\Delta r} = \frac{V(3) - V(1)}{3-1} = \frac{\frac{4}{3}\pi(3)^3 - \frac{4}{3}\pi(1)^3}{2} = \frac{36\pi - \frac{4}{3}\pi}{2}$$

$$= 18\pi - \frac{2}{3}\pi = \boxed{\frac{52\pi}{3} \text{ cm}^2}$$

$$\frac{d}{dx}(e^x) = e^x \qquad \frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(a^x) = a^x \ln a \qquad \frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

$$e \approx 2.7 \qquad \ln x = \log_e x$$

$$\boxed{\log_a b = c \iff a^c = b}$$

Power Rule:

$$\frac{d}{dx}[x^n] = nx^{n-1} \qquad \text{d/dx [c]=0}$$

Constant Multiple Rule:

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$$

Sum & Difference:

$$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$$

Product Rule:

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

Quotient Rule:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$$

Chain Rule:

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

Trig Functions:

$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\tan x] = \sec^2 x \qquad \frac{d}{dx}[\sec x] = \sec x \tan x$$

$$\frac{d}{dx}[\cos x] = -\sin x \qquad \frac{d}{dx}[\cot x] = -\csc^2 x \qquad \frac{d}{dx}[\csc x] = -\csc x \cot x$$

2.3 Product & Quotient Rules

$$[fg]' = \frac{d}{dx}[f(x)g(x)] \; f'(x)g(x) + f(x)g'(x)$$

$$\boxed{(fg)' = f'g + fg'}$$

$$\left[\frac{f}{g}\right]' = \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

"low dee high less high dee low,
draw the line and square below"

$$\boxed{\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}}$$

"low" = denominator

"dee high" = derivative of numerator

"less" = $-$

"high" = numerator

"dee low" = derivative of denominator

2.3

6. $g(x) = (\sqrt{x})(\sin x) = (x^{1/2})(\sin x)$

$g'(x) = (x^{1/2})'(\sin x) + (x^{1/2})(\sin x)'$

$\frac{dg}{dx} = \frac{d}{dx}[g(x)] = \boxed{\frac{1}{2}x^{-1/2}\sin x + (x^{1/2})\cos x}$

$= \frac{\sin x}{2\sqrt{x}} + \sqrt{x}\cos x$

12. $f(t) = \frac{\cos t}{t^3} = (\cos t)(t^{-3})$

$f'(t) = (-\sin t)(t^{-3}) + (\cos t)(-3t^{-4})$

$f'(t) = \frac{(t^3)(\cos t)' - (\cos t)(t^3)'}{(t^3)^2}$

$= \boxed{\frac{-t^3\sin t - 3t^2\cos t}{t^6}} = \frac{t^2(-t\sin t - 3\cos t)}{t^6}$

$= \boxed{\frac{-t\sin t - 3\cos t}{t^4}}$

26. $f(x) = \dfrac{x^3+3x+2}{x^2-1}$

Note: as a product,

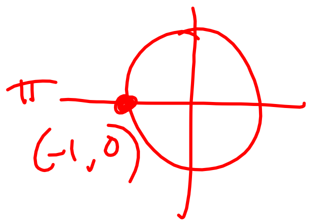
$f(x) = (x^3+3x+2)(x^2-1)^{-1}$

we don't know how to differentiate this yet so we have to use the quotient rule!

$$f'(x) = \boxed{\frac{(x^2-1)(3x^2+3) - (x^3+3x+2)(2x)}{(x^2-1)^2}}$$

Find the slope of the tangent line

$f(x) = 3x - \sin x \quad ; (\pi, 3\pi)$



$f'(x) = 3 - \cos x$

$m = f'(\pi) = 3 - \cos\pi = 3-(-1) = \boxed{4}$

Find the equation of the tangent line.

$f(x)=2x^3+\sin x-2x \quad ; (0,0)$

$m=f'(0) \qquad (x_1,y_1)$

$f'(x)=6x^2+\cos x-2$

$m=f'(0)=0+\cos 0-2$

$=1-2=-1$

$y-y_1=m(x-x_1)$

$y-0=-1(x-0)$

$\boxed{y=-x}$

2.1

32\. $f(x)=\dfrac{1}{x+1} \quad ; (0,1)=(x_1,y_1)$

$f'(x)=\dfrac{(x+1)(0)-1(1)}{(x+1)^2}=\dfrac{-1}{(x+1)^2}$

$m=f'(0)=\dfrac{-1}{(0+1)^2}=-1$

$y-1=-1(x-0)$

$\boxed{y=-x+1}$

Find f'(x)

2.2

43. $f(x) = \dfrac{x^3 - 3x^2 + 4}{x^2} = \dfrac{x^3}{x^2} - \dfrac{3x^2}{x^2} + \dfrac{4}{x^2}$

$= x - 3 + 4x^{-2}$

$f'(x) = \boxed{1 - 8x^{-3}} = 1 - \dfrac{8}{x^3} = \dfrac{x^3 - 8}{x^3}$

$= \dfrac{(x-2)(x^2+2x+4)}{x^3}$

$f(x) = \tan x = \dfrac{\sin x}{\cos x}$

$f'(x) = \dfrac{(\cos x)(\cos x) - \sin x(-\sin x)}{(\cos x)^2}$

$= \dfrac{\cos^2 x + \sin^2 x}{\cos^2 x} = \dfrac{1}{\cos^2 x} = \boxed{\sec^2 x}$

Power Rule:

$$\frac{d}{dx}[x^n] = nx^{n-1}$$

Product Rule:

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

Constant Multiple Rule:

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$$

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2.4 - The Chain Rule

$$[f(g(x))]' = f'(g(x)) \cdot g'(x) \cdot x'$$

$$[h(g(f(x)))]' = h'(g(f(x))) \cdot g'(f(x)) \cdot f'(x) \cdot x'$$

$$f(x) = \sin\left(x^5 - 3x^2\right)$$

$$f'(x) = \left[\cos\left(x^5 - 3x^2\right)\right] \cdot \left(5x^4 - 6x\right)$$

$$f(x) = \cos\left[5\sin(7x)\right]$$

$[5\sin 7x]'$

$$f'(x) = \boxed{-\sin\left[5\sin(7x)\right] \cdot \underbrace{5\cos(7x) \cdot 7}}$$

$$= -35\cos 7x \sin\left[5\sin 7x\right]$$