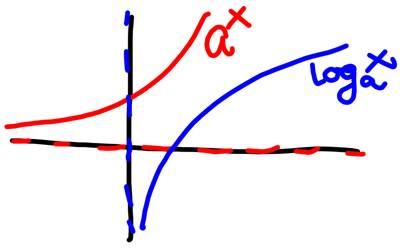
Ch 5 - Derivatives of Logarithmic and Exponential Functions

recall: $\ln x = \log_e x$

$e \approx 2.7$

$\log_2 8 = 3 \iff 2^3 = 8$

$\log_a b = c \iff a^c = b$



$y = 2^x$

x = the power to which we raise 2 to get y

= the # of times we multiply 2 by itself to get y

$= \log_2 y$

$$\frac{d}{dx}\left[2^x\right] = 2^x \ln 2$$

$$\frac{d}{dx}\left[a^x\right] = a^x \cdot \ln a$$

$$\frac{d}{dx}\left[\log_2 x\right] = \frac{1}{x \ln 2}$$

$$\frac{d}{dx}\left[\log_a x\right] = \frac{1}{x \cdot \ln a}$$

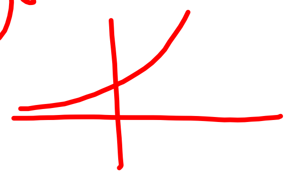
$$\frac{d}{dx}\left[\log_a u\right] = \frac{u'}{u \cdot \ln a}$$

$$\frac{d}{dx} \log_a f(x) = \frac{1}{f(x) \cdot \ln a} \cdot f'(x) = \frac{f'(x)}{f(x) \cdot \ln a}$$

$$\frac{d}{dx} a^{f(x)} = a^{f(x)} \ln a \cdot f'(x)$$

$$[e^x]' = e^x \cdot \ln e = e^x \log_e e = e^x$$

$$[e^x]' = e^x$$



$$[\ln x]' = \frac{1}{x \ln e} = \frac{1}{x}$$

Since the derivative of e^x is itself, this means that graphically, at every x-value, the slope of the tangent line at that point is exactly the y-coordinate.

$$f(x) = \ln\left[\sin\left(5x^3+2x\right)\right]$$

$$f'(x) = \boxed{\frac{1}{\sin(5x^3+2x)} \cdot \cos(5x^3+2x) \cdot (15x^2+2)}$$

$$= (15x^2+2)\cot(5x^3+2x)$$

$$f(x) = (\sec x)(5^{\sin x})$$

$$f'(x) = (\sec x \tan x)(5^{\sin x}) + (\sec x)(5^{\sin x} \ln 5 \cdot \cos x)$$

$$nx^{n-1} = [x^n]' \qquad [a^x]' = a^x \ln a$$

power fn exponential

$$f(x) = \sqrt[3]{\sin^2(\ln(4x^9))}$$

$$f(x) = 5^{\sqrt[3]{4\log_2(3x^2-4x)}} = 5^{(4\log_2(3x^2-4x))^{1/3}}$$

$$f'(x) = 5^{\sqrt[3]{4\log_2(3x^2-4x)}} \cdot \ln 5 \cdot \frac{1}{3}(4\log_2(3x^2-4x))^{-2/3} \cdot$$

$$\cdot 4 \cdot \frac{1}{(3x^2-4x)\ln 2} \cdot (6x-4)$$