Find $y', y'', y''', y^{(4)}, y^{(5)}, \dots, y^{(n)}$

$y = 5x^3 - 3x^2 + 2$

$y' = 15x^2 - 6x$

$y'' = 30x - 6$

$y''' = 30$

$y^{(4)} = 0$

$y^{(5)} = 0$

$y = x^6 + 2x^5 - 3x^4 + 2x - 5$

$y' = 6x^5 + 10x^4 - 12x^3 + 2$

$y'' = 30x^4 + 40x^3 - 36x^2$

$y''' = 120x^3 + 120x^2 - 72x$

$y^{(4)} = 360x^2 + 240x - 72$

$y^{(5)} = 720x + 240$

$y^{(6)} = 720$

$y^{(7)} = 0$

Find $y', y'', y''', y^{(4)}, y^{(5)}, \dots, y^{(n)}$

$y = 5x^3 - 3x^2 + 2$ $\qquad$ $y = x^6 + 2x^5 - 3x^4 + 2x - 5$

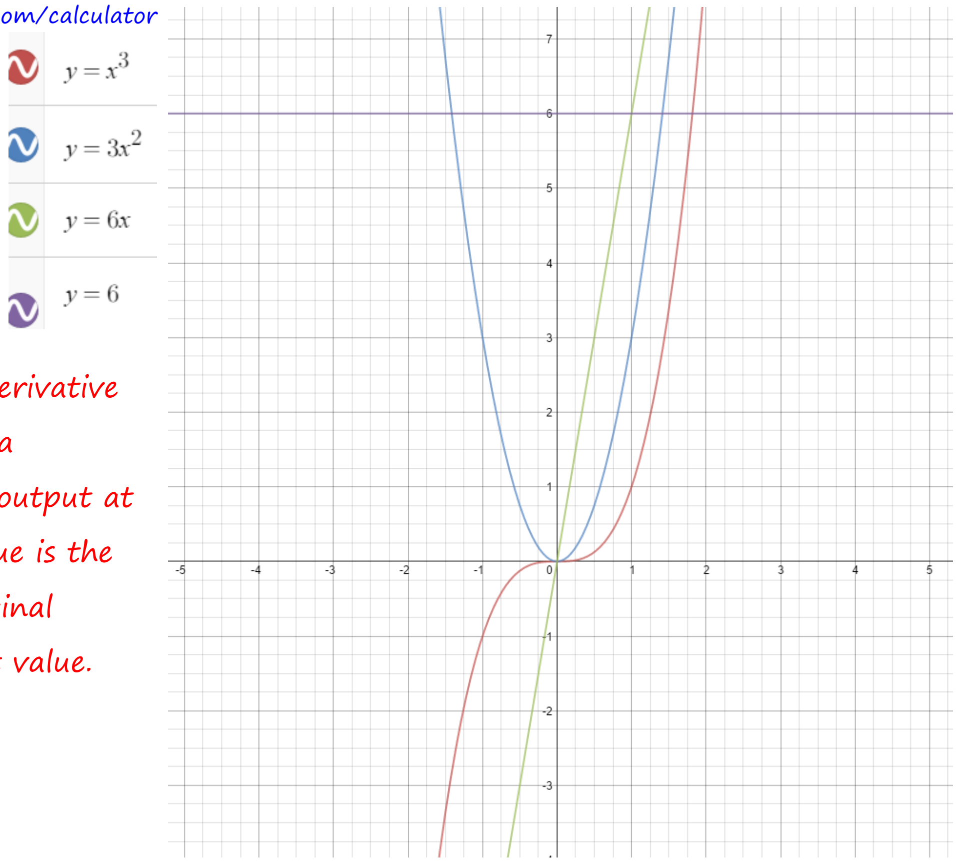
If $f(x)$ is a polynomial of degree n, then $f^{(n+1)}(x) = 0$.

If $f(x) = x^n$, then

$f^{(n)}(x) = n! = n(n-1)(n-2)\cdots 3 \cdot 2 \cdot 1$

$f(x) = 3x^9 - 15x^4 + 23x^{16} - 201x^7 - 3$ $\qquad$ $f^{(17)} = 0$

https://www.desmos.com/calculator



Note that the derivative of a function is a function whose output at a particular value is the slope of the original function at that value.

Power Rule:

$$\frac{d}{dx}[x^n] = nx^{n-1} \qquad \text{d/dx [c]=0}$$

Constant Multiple Rule:

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$$

Sum & Difference:

$$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$$

Product Rule:

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

Quotient Rule:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$$

Chain Rule:

$$\frac{d}{dx}[f(g(x))] = f'\big(g(x)\big)g'(x)$$

Trig Functions:

$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\tan x] = \sec^2 x \qquad \frac{d}{dx}[\sec x] = \sec x \tan x$$

$$\frac{d}{dx}[\cos x] = -\sin x \qquad \frac{d}{dx}[\cot x] = -\csc^2 x \qquad \frac{d}{dx}[\csc x] = -\csc x \cot x$$

$$f(x) = \cos\left(\sqrt{\tan^2 x - 2x}\right)$$

$$= \cos\left[\left([\tan x]^2 - 2x\right)^{1/2}\right]$$

$$f'(x) = -\sin\left(\sqrt{\tan^2 x - 2x}\right) \cdot \frac{1}{2}\left(\tan^2 x - 2x\right)^{-1/2} \cdot \left(2\tan x \cdot \sec^2 x - 2\right)$$

$$f(x) = \cot\left(5x^2 - 3x\right)$$

$$f'(x) = -\csc^2\left(5x^2 - 3x\right) \cdot \left(10x - 3\right)$$

$$f(x) = \sqrt[3]{\csc(4x)} = [\csc 4x]^{1/3}$$

$$f'(x) = \frac{1}{3}(\csc 4x)^{-2/3} \cdot (-\csc 4x \cot 4x) \cdot 4$$

$$f(x) = \frac{\sin 2x}{x^3} = (\sin 2x)(x^{-3})$$

$$f'(x) = \frac{x^3(\cos 2x \cdot 2) - 3x^2 \sin 2x}{(x^3)^2}$$

$$= \frac{\cancel{x^2}(2x\cos 2x - 3\sin 2x)}{x^{\cancel{6}4}}$$

$$= \frac{2x\cos 2x - 3\sin 2x}{x^4}$$

$$f(x) = \frac{x^2 \ln x}{\sin x}$$

$$f'(x) = \frac{\sin x\left[2x \cdot \ln x + x^2 \cdot \frac{1}{x}\right] - x^2 \ln x \cos x}{\sin^2 x}$$

$$f(x) = \sqrt[3]{\sin^2(\ln(4x^9))}$$

$$(x^m)^n = x^{mn}$$

$$= \left[\left(\sin[\ln(4x^9)]\right)^2\right]^{1/3}$$

$$= \left[\sin(\ln[4x^9])\right]^{2/3}$$

$$f'(x) = \frac{2}{\cancel{3}}\left[\sin(\ln[4x^9])\right]^{-1/3} \cdot \cos(\ln[4x^9]) \cdot \frac{1}{\cancel{4}x^{\cancel{9}\,1}} \cdot \overset{3}{\cancel{36}}x^{\cancel{8}}$$

$$= \frac{6\cos([\ln(4x^9)])}{x\sqrt[3]{\sin(\ln[4x^9])}}$$

2.4 The Chain Rule, cont.

18. $f(x) = -3\sqrt[4]{2-9x}$ $= -3(2-9x)^{1/4}$

$f'(x) = -\frac{3}{4}(2-9x)^{-3/4} \cdot (-9)$

$\boxed{(a+b)^2 = a^2+2ab+b^2}$

32. $h(t) = \left(\frac{t^2}{t^3+2}\right)^2$ $= \frac{t^4}{t^6+4t^3+4}$

$h'(t) = \frac{(t^6+4t^3+4)(4t^3) - t^4(6t^5+12t^2)}{(t^6+4t^3+4)^2}$

$\boxed{h'(t) = 2\left(\frac{t^2}{t^3+2}\right) \cdot \left(\frac{(t^3+2)(2t) - t^2(3t^2)}{(t^3+2)^2}\right)}$

50. $h(x) = \sec x^2$
$= \sec(x^2)$

$h'(x) = \sec x^2 \tan x^2 \cdot 2x$

60. $g(t) = 5\cos^2 \pi t$ $= 5(\cos \pi t)^2$

$2\sin x \cos x = \sin 2x$

$g'(t) = 10\cos \pi t(-\sin \pi t) \cdot \pi$

$= -10\pi \sin\pi t \cos \pi t = -5\pi \sin 2\pi t$

66. $y = \sin\sqrt[3]{x} + \sqrt[3]{\sin x}$

$= \sin(x^{1/3}) + (\sin x)^{1/3}$

$y' = \cos(x^{1/3}) \cdot \frac{1}{3}x^{-2/3} + \frac{1}{3}(\sin x)^{-2/3} \cdot \cos x$

5.4

46. $g(t) = e^{-3/t^2}$ $= e^{-3t^{-2}}$

$g'(t) = e^{-3t^{-2}} \cdot (6t^{-3})$

$\boxed{\frac{c}{ax^n} = \frac{c}{a} \cdot x^{-n}}$

48. $y = \ln\left(\frac{1+e^x}{1-e^x}\right) = \ln(1+e^x) - \ln(1-e^x)$

$y' = \frac{e^x}{1+e^x} + \frac{e^x}{1-e^x}$

$\log_a MN = \log_a M + \log_a N$

$\log_a \frac{M}{N} = \log_a M - \log_a N$

$\log_a M^p = p\log_a M$

58. $y = \ln e^x = x$

$y' = 1$

$(\log_a M^p)^2 \neq \log_a (M^p)$

$\log_a a^x = x$

$a^{\log_a x} = x$

<u>5.5</u>

46. $f(t) = \frac{3^{2t}}{t}$

$f'(t) = \frac{t \cdot 3^{2t} \ln 3 \cdot 2 - 3^{2t}}{t^2}$

$[a^x]' = a^x \ln a$

$[x^n]' = nx^{n-1}$

54. $y = \log_{10} \frac{x^2-1}{x} = \log_{10}(x^2-1) - \log_{10} x$

$y' = \frac{2x}{(\ln 10)(x^2-1)} - \frac{1}{x \ln 10}$

$[x^n]' = nx^{n-1}$

$[cf(x)]' = cf'(x)$

$[f(x) \pm g(x)]' = f'(x) \pm g'(x)$

$[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$

$\left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$

$[f(g(x))]' = f'(g(x)) \cdot g'(x)$

$[e^x]' = e^x$

$[a^x]' = a^x \ln a$

$[\ln x]' = \frac{1}{x}$

$[\log_a x]' = \frac{1}{x \ln a}$

$[\sin x]' = \cos x$

$[\cos x]' = -\sin x$

$[\tan x]' = \sec^2 x$

$[\cot x]' = -\csc^2 x$

$[\sec x]' = \sec x \tan x$

$[\csc x]' = -\csc x \cot x$

$[\arcsin x]' = \frac{1}{\sqrt{1-x^2}}$

$[\arctan x]' = \frac{1}{1+x^2}$

$[\text{arcsec}\, x]' = \frac{1}{|x|\sqrt{x^2-1}}$

$[\arccos x]' = \frac{-1}{\sqrt{1-x^2}}$

$[\text{arccot}\, x]' = \frac{-1}{1+x^2}$

$[\text{arccsc}\, x]' = \frac{-1}{|x|\sqrt{x^2-1}}$