$$f(x) = 5\sin^2\left(\ln\left[\sqrt[3]{\frac{x^3-1}{x-1}}\right]\right) + 5\cos^2\left(\frac{\ln(x^2+x+1)}{3}\right)$$

$$\frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{x-1}} \quad 1/3$$

$$\ln\left(x^2+x+1\right)$$

$$= 5\left(\sin^2\left(\frac{\ln(x^2+x+1)}{3}\right) + \cos^2\left(\frac{\ln(x^2+x+1)}{3}\right)\right)$$

$$= 5$$

$$f'(x) = 0$$

Instantaneous rate of change of a function $f(x)$ when $x = c$ is $f'(c)$ <-- slope of tangent line through a single point

Average rate of change of a function $f(x)$ on the interval $[a, b]$ is $\frac{f(b)-f(a)}{b-a}$ <-- slope of secant line through two points

Given a position function $s(t) = gt^2 + v_0 t + s_0$,

Since velocity is the rate of change of position,

The instantaneous velocity at time $t = c$ is $s'(c)$

The average velocity on the interval $[a, b]$ is $\frac{s(b)-s(a)}{b-a}$

speed v. velocity

$+$ $\quad$ $+/-$

$|v|$

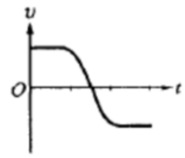
The Derivative

The slope of the tangent line to the graph of f at the point $(c, f(c))$ is given by:

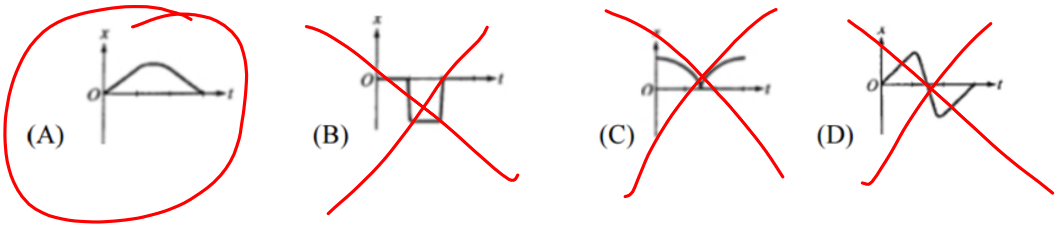
$$m = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(c+\Delta x) - f(c)}{\Delta x}$$

The derivative of f at x is given by

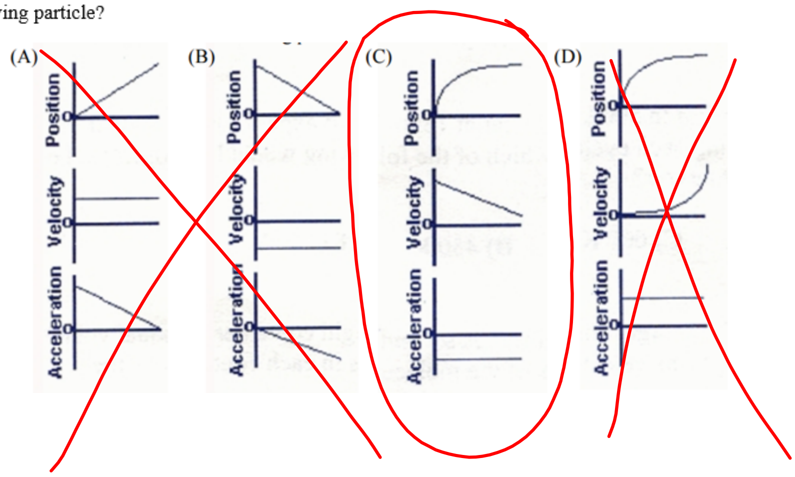
$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

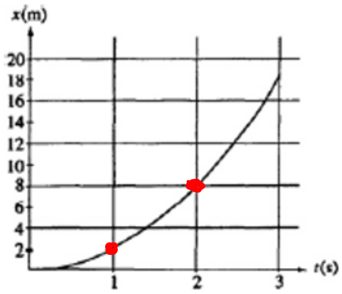


The graph above shows velocity v versus time t for an object in linear motion. Which of the following is a possible graph of position x versus time t for this object?



Which of the following sets of graphs below might be the corresponding graphs of position, velocity, and acceleration vs time for a moving particle?





The graph above represents position x versus time t for an object being acted on by a constant force. The average speed during the interval between 1 s and 2 s is most nearly

(A) 2 m/s *(B) 4 m/s* *(C) 5 m/s* *(D) 6 m/s*

$$\frac{x(2)-x(1)}{2-1} = \frac{8-2}{1} = 6$$

Consider the motion of an object given by the position vs. time graph shown below. For what time(s) is the speed of the object greatest?

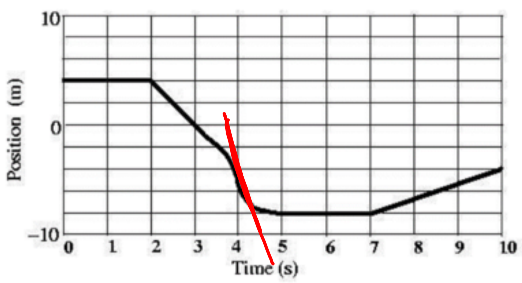
(A) At all times from t = 0.0 s→ t = 2.0 s

(B) At time t = 3.0 s

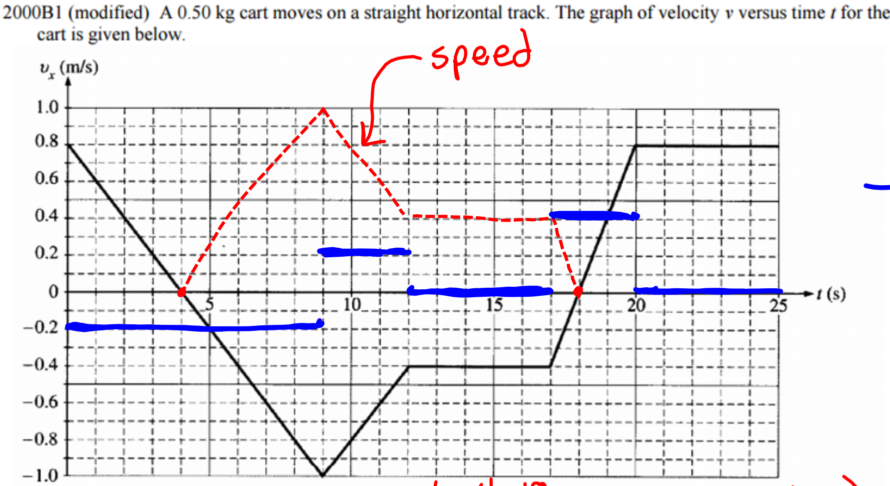
(C) At time t = 4.0 s

(D) At all times from t = 5.0 s → t = 7.0 s

(E) At time t = 8.5 s



2000B1 (modified) A 0.50 kg cart moves on a straight horizontal track. The graph of velocity v versus time t for the cart is given below.



— acceleration

a. Indicate every time t for which the cart is at rest. $t = 4, 18$

b. Indicate every time interval for which the speed (magnitude of velocity) of the cart is increasing. $(4,9) \cup (18,20)$

c. ~~Determine the horizontal position x of the cart at t = 9.0 s if the cart is located at x = 2.0 m at t = 0.~~

d. On the axes below, sketch the acceleration a versus time t graph for the motion of the cart from t = 0 to t = 25 s.

5.8

44. $f(x) = \operatorname{arcsec} 2x$

$\boxed{\arcsin x = \sin^{-1} x}$

$\frac{d}{dx}[\arcsin x] = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}[\arctan x] = \frac{1}{1+x^2}$

$\frac{d}{dx}[\operatorname{arcsec} x] = \frac{1}{|x|\sqrt{x^2-1}}$

$\frac{d}{dx}[\arccos x] = \frac{-1}{\sqrt{1-x^2}}$

$\frac{d}{dx}[\operatorname{arccot} x] = \frac{-1}{1+x^2}$

$\frac{d}{dx}[\operatorname{arccsc} x] = \frac{-1}{|x|\sqrt{x^2-1}}$

$$f'(x) = \frac{1}{|2x|\sqrt{(2x)^2-1}} \cdot 2 = \frac{1}{|x|\sqrt{4x^2-1}}$$

48. $h(x) = x^2 \arctan x$

$$h'(x) = 2x \cdot \arctan x + x^2 \cdot \frac{1}{1+x^2}$$

52. $y = \ln(t^2+4) - \frac{1}{2}\arctan\frac{t}{2}$

$$\frac{t}{2} = \frac{1}{2}t$$

$$y' = \frac{2t}{t^2+4} - \frac{1}{2} \cdot \frac{1}{1+\left(\frac{t}{2}\right)^2} \cdot \frac{1}{2}$$

$$= \frac{2t}{t^2+4} - \frac{1}{4\left(1+\frac{t^2}{4}\right)} = \frac{2t}{t^2+4} - \frac{1}{4+t^2}$$

$$= \frac{2t-1}{t^2+4}$$

56. $y = x \arctan 2x - \frac{1}{4}\ln(1+4x^2)$

$y' = \arctan 2x + x \cdot \frac{1}{1+(2x)^2} \cdot 2 - \frac{1}{4} \cdot \frac{1}{1+4x^2} \cdot 8x$

$\frac{2x}{1+4x^2} - \frac{2x}{1+4x^2}$

$= \boxed{\arctan 2x}$

$\frac{d}{dx}[\arcsin x] = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}[\arctan x] = \frac{1}{1+x^2}$

$\frac{d}{dx}[\text{arcsec}\, x] = \frac{1}{|x|\sqrt{x^2-1}}$

$\frac{d}{dx}[\arccos x] = \frac{-1}{\sqrt{1-x^2}}$

$\frac{d}{dx}[\text{arccot}\, x] = \frac{-1}{1+x^2}$

$\frac{d}{dx}[\text{arccsc}\, x] = \frac{-1}{|x|\sqrt{x^2-1}}$

<u>5.4 – Find the second derivative</u>

80. $f(x) = \frac{1}{x-2} = (x-2)^{-1}$

$f'(x) = -(x-2)^{-2} = \frac{-1}{(x-2)^2}$

$f''(x) = 2(x-2)^{-3} = \boxed{\frac{2}{(x-2)^3}}$

Find the second derivative.

82. $f(x) = \sec^2 \pi x = [\sec(\pi x)]^2$

$f'(x) = 2\sec\pi x \cdot \sec\pi x \tan\pi x \cdot \pi$

$= [2\pi \sec^2\pi x][\tan\pi x]$

$f''(x) = 2\pi \cdot (2\pi \sec^2\pi x \tan\pi x) + 2\pi\sec^2\pi x \cdot \sec^2\pi x \cdot \pi$

$= \boxed{4\pi^2 \sec^2\pi x \tan\pi x + 2\pi^2\sec^4\pi x}$