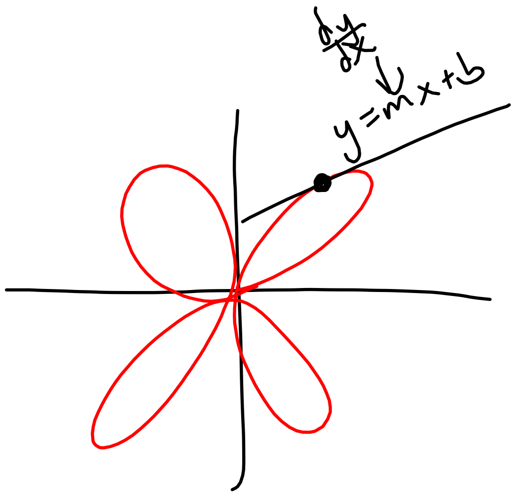
What happens if...

$$x^2 y + y^2 x = -2$$

how to find y'?



2.5 Implicit Differentiation

☆ $y = f(x)$

y is a function of x

$\frac{d}{dx}[x] = 1$; $\frac{d}{dx}[y] = y'$

$$\frac{d}{dx}[f(y)] = f'(y) \cdot y'$$

$\frac{dy}{dx}$

6. $x^2y + y^2x = -2$

$$\frac{d}{dx}\left[x^2y + y^2x\right] = \frac{d}{dx}\left[-2\right]$$

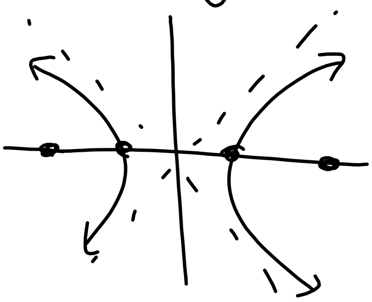
$$2xy + x^2y' + 2y\cdot y'x + y^2\cdot 1 = 0$$

$$x^2y' + 2yy'x = -y^2 - 2xy$$

$$y'(x^2 + 2xy) = -y^2 - 2xy$$

$$y' = \frac{-y^2 - 2xy}{x^2 + 2xy}$$

2. $x^2 - y^2 = 16$

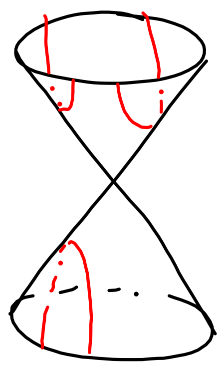


$$\frac{d}{dx}\left[x^2 - y^2\right] = \frac{d}{dx}\left[16\right]$$

$$2x - 2yy' = 0$$

$$2x = 2yy'$$

$$\frac{2x}{2y} = y' \Rightarrow y' = \frac{x}{y}$$



8. $\sqrt{xy} = x - 2y$

$$\frac{d}{dx}\left[(xy)^{1/2}\right] = \frac{d}{dx}\left[x - 2y\right]$$

$$\frac{1}{2}(xy)^{-1/2} \cdot (1 \cdot y + xy') = 1 - 2y'$$

$$\frac{y + xy'}{2\sqrt{xy}} = 1 - 2y'$$

$$y + xy' = 2\sqrt{xy}\,(1 - 2y')$$

$$y + xy' = 2\sqrt{xy} - 4y'\sqrt{xy}$$

$$xy' + 4y'\sqrt{xy} = 2\sqrt{xy} - y$$

$$y' = \frac{2\sqrt{xy} - y}{x + 4\sqrt{xy}}$$

10. $(2\sin x)(\cos y) = 1$

$\frac{d}{dx}$

$$2\cos x \cos y + 2\sin x(-\sin y)y' = 0$$

$$2\cos x \cos y = 2\sin x \sin y \cdot y'$$

$$\cot x \cot y = y'$$

12. $(\sin\pi x + \cos\pi y)^2 = 2$

$$(2\sin\pi x + 2\cos\pi y)\cdot(\pi\cos\pi x - \pi y'\sin\pi y) = 0$$

$$2\pi\sin\pi x\cos\pi x - \overbrace{2\pi y'\sin\pi x\sin\pi y} + 2\pi\cos\pi x\cos\pi y - \overbrace{2\pi y'\sin\pi y\cos\pi y}^{=0}$$

$$\frac{2\pi\sin\pi x\cos\pi x + 2\pi\cos\pi x\cos\pi y}{2\pi\sin\pi x\sin\pi y + 2\pi\sin\pi y\cos\pi y} = y'$$

$$\frac{\cancel{2\pi}\cos\pi x(\sin\pi x \cancel{+\cos\pi y})}{\cancel{2\pi}\sin\pi y(\sin\pi x \cancel{+\cos\pi y})} ? = y'$$

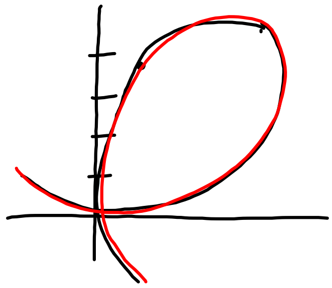
$$\frac{\cos\pi x}{\sin\pi y} = y'$$

16. $x = \sec\frac{1}{y}$

32. Folium of Descartes

$x^3+y^3-6xy=0$

find the slope of the tangent line @ $\left(\frac{4}{3}, \frac{8}{3}\right)$



40. Find y'' in terms of x & y.

$y^2=4x$

$2yy'=4$

$y'=\frac{4}{2y}=2y^{-1}$

$y''=-2y^{-2}y'=\frac{-2y'}{y^2}=\frac{-2\left(\frac{2}{y}\right)}{y^2}=\boxed{\frac{-4}{y^3}}$