5.8-ish

$$\frac{d}{dx}[\arcsin x] = \frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}[\arctan x] = \frac{1}{1+x^2}$$
$$\frac{d}{dx}[\text{arcsec}\, x] = \frac{1}{|x|\sqrt{x^2-1}}$$
$$\frac{d}{dx}[\arccos x] = \frac{-1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}[\text{arccot}\, x] = \frac{-1}{1+x^2}$$
$$\frac{d}{dx}[\text{arccsc}\, x] = \frac{-1}{|x|\sqrt{x^2-1}}$$

$f(x) = \arcsin(3x)$

$$f'(x) = \frac{1}{\sqrt{1-(3x)^2}} \cdot 3$$

$f(x) = \arctan(\ln(2x))$

$$f'(x) = \frac{1}{1+(\ln 2x)^2} \cdot \frac{1}{2x} \cdot 2$$

$f(x) = \cot\left(5^{\arcsin(4x^3)}\right)$

$$f'(x) = -\csc^2\left(5^{\arcsin 4x^3}\right) \cdot 5^{\arcsin 4x^3} \cdot \ln 5 \cdot \frac{1}{\sqrt{1-(4x^3)^2}} \cdot 12x^2$$

$f(x) = 5\sqrt{3\csc^2(\log_4(\text{arccsc}(7x^9)))}$

$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

$$= 5\left[3\left(\csc\left[\log_4\left(\text{arccsc}\left[7x^9\right]\right)\right]\right)^2\right]^{1/2}$$

$$= 5\sqrt{3}\,\csc\left[\log_4\left(\text{arccsc}\left[7x^9\right]\right)\right]$$

$$f'(x) = -5\sqrt{3}\,\csc\left[\log_4\left(\text{arccsc}\left[7x^9\right]\right)\right]\cot\left[\log_4\left(\text{arccsc}\left[7x^9\right]\right)\right] \cdot \frac{1}{\ln 4 \cdot \text{arccsc}\left[7x^9\right]} \cdot \frac{-1}{|7x^9|\sqrt{(7x^9)^2-1}} \cdot 63x^8$$

$$f(x) = \tan\left[\log_2\left(\sin\left[3^{5x}\right]\right)\right]$$

$$f(x) = -2^{\ln(\arctan(7x^5 - \cos 3x))} \neq (-2)^{\ln(\sim\sim)}$$

$$= (-1)\,2^{\ln \sim\sim}$$

$$f'(x) = -2^{\ln \sim} \cdot \ln 2 \cdot \frac{1}{\sim} \sim$$

16. $x = \sec \frac{1}{y}$

$$\frac{d}{dx}[x] = \frac{d}{dx}\left[\sec\frac{1}{y}\right]$$

$$1 = \sec\frac{1}{y}\tan\frac{1}{y} \cdot \frac{-1}{y^2} \cdot \frac{dy}{dx}$$

$$\frac{1}{\sec\frac{1}{y}\tan\frac{1}{y}\cdot\frac{-1}{y^2}} = \frac{dy}{\cancel{dx}} = \boxed{-y^2\cos\frac{1}{y}\cot\frac{1}{y}}$$

$$\frac{1}{x} = x^{-1}$$

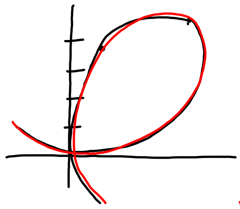
$$\left(\frac{1}{x}\right)' = \left(x^{-1}\right)' = -x^{-2}$$

$$= \frac{-1}{x^2}$$

32. Folium of Descartes

$x^3+y^3-6xy=0$

find the slope of the tangent line @ $\left(\frac{4}{3}, \frac{8}{3}\right)$



$$\frac{d}{dx}\left[x^3+y^3\right] = \frac{d}{dx}\left[(6x)y\right]$$

$$\underbrace{3x^2+3y^2\cdot\frac{dy}{dx}}=6y+\underbrace{6x\cdot\frac{dy}{dx}}$$

$$\frac{dy}{dx}\left(3y^2-6x\right)=6y-3x^2$$

$$\frac{dy}{dx}=\frac{\cancel{3}(2y-x^2)}{\cancel{3}(y^2-2x)}$$

$$\left.\frac{dy}{dx}\right|_{(4/3,\,8/3)}=\frac{2(8/3)-(4/3)^2}{(8/3)^2-2(4/3)}=\frac{\frac{16}{3}-\frac{16}{9}}{\frac{64}{9}-\frac{8}{3}}\cdot\frac{9}{9}=\frac{48-16}{64-24}$$

$$=\frac{32}{40}=\boxed{\frac{4}{5}}$$

2.6 Related Rates

18. $V=\frac{4}{3}\pi r^3$

$\frac{dr}{dt}=2\text{ in/min}$

$\frac{dV}{dt}=?$ when $r=6\text{ in}$

$$\frac{d}{dt}[V]=\frac{d}{dt}\left[\frac{4}{3}\pi r^3\right]$$

$$\frac{dV}{dt}=4\pi r^2\cdot\frac{dr}{dt}=4\pi(6)^2\cdot 2=\boxed{288\pi\ \text{in}^3/\text{min}}$$

$$=4\pi\left(6^2\text{in}^2\right)\cdot\frac{2\text{in}}{\text{min}}$$

22. $V = \frac{1}{3}\pi r^2 h$

$= \frac{\pi}{3} r^2 (3r)$

$\frac{dV}{dt} = ?$ when $r = 6$ in

$\frac{dr}{dt} = 2 \frac{\text{in}}{\text{min}}$

$h = 3r$

$$\frac{d}{dt}[V] = \frac{d}{dt}\left[\left(\frac{\pi}{3} r^2\right) h\right]$$

$$\frac{dV}{dt} = \frac{2\pi}{3} r \cdot \frac{dr}{dt} \cdot h + \frac{\pi}{3} r^2 \cdot \frac{dh}{dt}$$

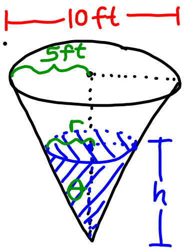
$$\frac{d}{dt}[h] = \frac{d}{dt}[3r]$$

$$\frac{dh}{dt} = \frac{3dr}{dt}$$

$$V = \pi r^3$$

$$\frac{dV}{dt} = 3\pi r^2 \cdot \frac{dr}{dt} = 3\pi(6)^2 \cdot 2 = \boxed{216\pi \frac{\text{in}^3}{\text{min}}}$$

24.



$\frac{dV}{dt} = 10 \text{ ft}^3/\text{min}$

$\frac{dh}{dt} = ?$ when $h = 8$ ft

$V = \frac{1}{3}\pi r^2 h$

$\frac{r}{h} = \frac{5}{12} \Rightarrow r = \frac{5h}{12}$

$$V = \frac{\pi}{3}\left(\frac{5h}{12}\right)^2 \cdot h$$

$$V = \frac{\pi}{3} \cdot \frac{25}{144} h^3$$

$$\frac{dV}{dt} = \frac{\pi}{\cancel{3}} \cdot \frac{25}{144} \cdot \cancel{3} h^2 \cdot \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{dV}{dt} \cdot \frac{144}{25\pi h^2} = 10 \cdot \frac{144}{25\pi \cdot 8^2}$$

$$\frac{dh}{dt} = \frac{\cancel{5} \cdot \cancel{2} \cdot \cancel{4} \cdot \cancel{4} \cdot 9}{\cancel{5} \cdot 5 \cdot \cancel{8} \cdot \cancel{4} \cdot 2\pi} = \boxed{\frac{9}{10\pi} \frac{\text{ft}}{\text{min}}}$$