3.3-3.4 Increasing, Decreasing, Concavity, and the 1st and 2nd Derivative Tests

What do *f′* and *f″* tell us about *f*?

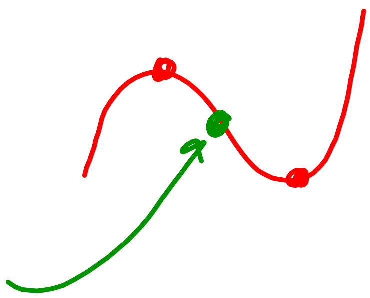
Recall that *f′* is the rate of change or slope of *f*,
f″ is the slope or rate of change of *f′*.

f'	f
+	↗ increasing
−	↘ decreasing

f''	f'	f
+	↗ increasing	concave up
−	↘ decreasing	concave down

f′(x)=0 when f has a relative maximum or minimum.
These x-values (and those where f′(x) is undefined) are called critical numbers.

f″(x)=0 when f changes concavity.
The points where concavity changes are called inflection points.



To solve problems involving concavity, increasing/decreasing, etc., we should recall how to solve polynomial inequalities.

$$\frac{(x+2)(x-3)}{x+4} \geq 0$$

$$\boxed{(-4,-2] \cup [3,\infty)}$$

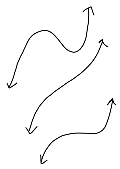
$f(-5)$ $f(-3)$ $f(0)$ $f(4)$

$-$ -4 $+$ -2 $-$ 3 $+$

- Find all critical numbers and state the open intervals on which f is increasing and/or decreasing.
- Find all inflection points and state the open intervals on which f is concave up and/or concave down.
- Use these results to determine all relative and absolute extrema.

3.3

16. $f(x)=x^3-6x^2+15$



$f'(x)=3x^2-12x$

$3x(x-4)=0$

critical#'s : $x=0,4$

$f'(-1)$ $f'(1)$ $f'(5)$

$+$ 0 $-$ 4 $+$

$\boxed{\begin{array}{l} f \text{ is increasing on } (-\infty,0)\cup(4,\infty) \\ f \text{ is decreasing on } (0,4) \\ f \text{ has a relative maximum @ } (0,15) \\ f \text{ has a relative minimum @ } (4,-17) \end{array}}$

$f''(x)=6x-12$

$6(x-2)=0$

$f''(0)$ $f''(3)$

$-$ 2 $+$

$\boxed{\begin{array}{l} f \text{ is concave down on } (-\infty,2) \\ f \text{ is concave up on } (2,\infty) \\ f \text{ has an inflection point @ } (2,-1) \end{array}}$

3.4

16. $f(x) = x^3(x-4) = x^4 - 4x^3$

$f'(x) = 4x^3 - 12x^2$

$0 = 4x^2(x-3)$

$f'(-1)$ $f'(1)$ $f'(4)$

$-$ 0 $-$ 3 $+$

f is decreasing on $(-\infty, 3)$ & increasing on $(3, \infty)$
f has a relative minimum @ $(3, -27)$

$f''(x) = 12x^2 - 24x$

$0 = 12x(x-2)$

$f''(-1)$ $f''(1)$ $f''(3)$

$+$ 0 $-$ 2 $+$

f is concave up on $(-\infty, 0) \cup (2, \infty)$ & concave down on $(0, 2)$
f has inflection points @ $(0, 0)$ & $(2, -16)$

3.3

30. $f(x) = \dfrac{x+3}{x^2}$

$$f'(x) = \frac{x^2(1) - (x+3)(2x)}{(x^2)^2} = \frac{x(x - 2(x+3))}{x^4} = \frac{-x-6}{x^3} = \frac{-1(x+6)}{x^3}$$

critical #'s: $-6, 0$

$f'(-7)$ $f'(-1)$ $f'(1)$

$-$ -6 $+$ 0 $-$

f is decreasing on $(-\infty, -6) \cup (0, \infty)$
f is increasing on $(-6, 0)$
f has a relative min @ $(-6, -\frac{1}{12})$
& no relative maximum

$$f''(x) = \frac{(x^3)(-1) - (-x-6)(3x^2)}{(x^3)^2} = \frac{-x^3 + 3x^3 + 18x^2}{x^6}$$

$$= \frac{2x^3 + 18x^2}{x^6} = \frac{2x^2(x+9)}{x^6} = \frac{2(x+9)}{x^4}$$