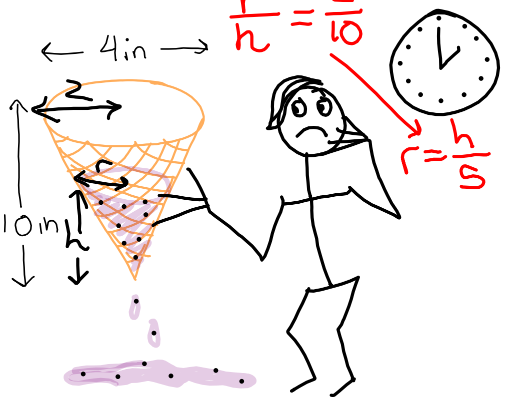
1. A jumbo waffle cone from Sarah's Tasty Ice Cream Shoppe is 10 inches tall and has a 4 inch diameter at the top of the cone. Yesterday, my cone had a leak! Instead of eating it super fast, I decided to compare the rate of change of volume of ice cream to the rate of change of height of ice cream in the cone. How fast is the ice cream leaking out (in cubic inches per minute) when there are 5 inches of ice cream in the cone, if the height of ice cream in the cone is changing at a rate of 1 inch every 5 minutes?



$\frac{r}{h} = \frac{2}{10}$

$r = \frac{h}{5}$

$\frac{dV}{dt} = ?$ when $h = 5$ in

$\frac{dh}{dt} = \frac{-1\text{ in}}{5\text{ min}}$

$V = \frac{\pi}{3} r^2 h$

$V = \frac{\pi}{3}\left(\frac{h}{5}\right)^2 \cdot h$

$V = \frac{\pi}{75} h^3$

$$\frac{dV}{dt} = \frac{\pi}{25} h^2 \cdot \frac{dh}{dt} = \frac{\pi}{25}(5)^2 \cdot \frac{-1}{5} = \boxed{-\frac{\pi}{5}\ \frac{\text{in}^3}{\text{min}}}$$

7. Find the derivative of f with respect to x.

$f(x) = e^{\arctan(2x+5)}$

$$f'(x) = e^{\arctan(2x+5)} \cdot \frac{1}{1+(2x+5)^2} \cdot 2$$

8. Find the derivative of f with respect to x.

$f(x) = (\operatorname{arccsc} x)(\ln(\tan(2x)))$

$$f'(x) = \frac{-1}{|x|\sqrt{x^2-1}} \cdot \ln(\tan(2x)) + \operatorname{arccsc} x \cdot \frac{1}{\tan 2x} \cdot \sec^2 2x \cdot 2$$

9. Find y' implicitly in terms of x and y.

$x^2y + 3xy^3 = 5x^3y^2$

$$2xy + x^2y' + 3y^3 + 9xy^2y' = 15x^2y^2 + 10x^3yy'$$

$$y' = \frac{15x^2y^2 - 2xy - 3y^3}{x^2 + 9xy^2 - 10x^3y}$$

10. Find y' implicitly in terms of x and y.

$\cos x + \sin y = \tan(xy)$

$$-\sin x + y'\cos y = \sec^2(xy)\cdot(y + xy')$$

$$y'\cos y - xy'\sec^2(xy) = y\sec^2(xy) + \sin x$$

$$y' = \frac{y\sec^2(xy) + \sin x}{\cos y - x\sec^2(xy)}$$

2. $x^3 + y^2 = 10$

a. Find y' in terms of x and y.

b. Find y'' in terms of x and y.

$$3x^2 + 2yy' = 0$$

$$\boxed{y' = \frac{-3x^2}{2y}}$$

$$y'' = \frac{(2y)(-6x) - (-3x^2)(2y')}{(2y)^2}$$

$$y'' = \frac{-12xy + 6x^2\left(\frac{-3x^2}{2y}\right)}{4y^2}$$

3. $f(x) = (x-3)^2(x+2)$

a. Find the critical numbers of the function. $-\frac{1}{3}, 3$

b. Find the absolute maximum and absolute minimum (if any) on the closed interval $[-1,1]$.

c. Find all open intervals on the function's domain on which it is increasing or decreasing.

$$f'(x) = 2(x-3)(x+2) + (x-3)^2$$
$$= 2x^2 - 2x - 12 + x^2 - 6x + 9$$
$$0 = 3x^2 - 8x - 3$$
$$0 = (3x+1)(x-3)$$

critical #'s: $-\frac{1}{3}, 3$

f is increasing on $(-\infty, -\frac{1}{3}) \cup (3, \infty)$ & decreasing on $(-\frac{1}{3}, 3)$

$f(-1) = 16$

$f(-\frac{1}{3}) = \frac{500}{27} \leftarrow$ abs max

$f(1) = 12 \leftarrow$ abs min

$f'(-1)$, $f'(0)$, $f'(4)$

$-\frac{1}{3}$ 3

$+$ $-$ $+$

5. Find the open intervals on which the function is increasing or decreasing and locate all relative extrema. $y = \frac{x^2}{x^2-9}$

6. Find the points of inflection and discuss concavity of the graph of the function. $f(x) = x^3(x-4)$

$= x^4 - 4x^3$

$f'(x) = 4x^3 - 12x^2$

$f''(x) = 12x^2 - 24x$

$0 = 12x(x-2)$

$f''(-1)$ $f''(1)$ $f''(3)$

$+$ 0 $-$ 2 $+$

f is concave up on $(-\infty, 0) \cup (2, \infty)$

& concave down on $(0, 2)$

f has inflection points @ $(0, 0)$ & $(2, -16)$

4. $f(x) = (x+3)^2(x-1)$ $= (x^2+6x+9)(x-1) = x^3+6x^2+9x-x^2-6x-9$

a. Determine if Rolle's Theorem can be applied on the closed interval $[-3,1]$, and if so, find all values of c in $(-3,1)$ guaranteed by that theorem such that $f'(c) = 0$.

b. Determine if the Mean Value Theorem can be applied on the closed interval $[-2,2]$, and if so, find all values of c in $(-2,2)$ guaranteed by that theorem such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

f cts on $[a,b]$ & diff on (a,b) $\Rightarrow$ MVT applies

these plus $f(a) = f(b)$ $\Rightarrow$ Rolle's Thm applies

$f(x) = x^3 + 5x^2 + 3x - 9$

$f'(x) = 3x^2 + 10x + 3$

$0 = (3x+1)(x+3)$

(a) $x = \cancel{-3}, -\frac{1}{3}$

$\frac{f(2)-f(-2)}{2-(-2)} = \frac{25-(-3)}{4} = \frac{28}{4} = 7$

(b) $3x^2 + 10x + 3 = 7$

$3x^2 + 10x - 4 = 0$

$x = \frac{-10 \pm \sqrt{10^2 - 4(3)(-4)}}{2(3)}$