3.5 Limits at Infinity

$\lim_{x\to\infty} f(x)$ (end behavior)

correspond exactly with horizontal & oblique asymptotes

$$f(x) = \frac{5x^2-3x+4}{2x^2+5x} \approx \frac{5x^2}{2x^2} = \frac{5}{2}$$

horizontal asymptote @ $y = \frac{5}{2}$

$$\lim_{x\to\pm\infty} f(x) = \frac{5}{2}$$

$$f(x) = \frac{2x-4}{3x^4} \approx \frac{2x}{3x^4} = \frac{2}{3}\cdot\frac{1}{x^3} \longrightarrow 0$$

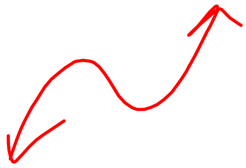
$\frac{1}{10}, \frac{1}{100}, \frac{1}{1000}, \frac{1}{1000000}$

$-\frac{1}{10}, -\frac{1}{100}, -\frac{1}{1000}, -\frac{1}{1000000}$

horizontal asymptote @ $y=0$

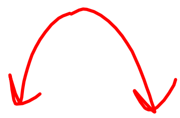
$$\lim_{x\to\pm\infty} f(x) = 0$$

$$f(x)=\frac{2x^7-4x^3-2}{5x^4+1}\approx\frac{2x^7}{5x^4}=\frac{2}{5}x^3$$

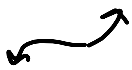
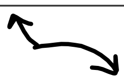
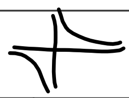
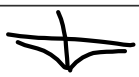


$$\lim_{x\to-\infty}f(x)=-\infty\ ;\ \lim_{x\to\infty}f(x)=\infty$$

$$f(x)=\frac{2-7x^3+2x}{1+x}\approx\frac{-7x^3}{x}=-7x^2$$



$$\lim_{x\to-\infty}f(x)=-\infty\ ;\ \lim_{x\to\infty}f(x)=-\infty$$

Ratio of lead terms of $f(x)$	Picture of graph end behavior	$\lim\limits_{x\to-\infty} f(x)$	$\lim\limits_{x\to\infty} f(x)$
$+x^{even}$		∞	∞
$-x^{even}$		$-\infty$	$-\infty$
$+x^{odd}$		$-\infty$	∞
$-x^{odd}$		∞	$-\infty$
c		c	c
$+\frac{1}{x^{odd}}$		0	0
$-\frac{1}{x^{odd}}$		0	0
$+\frac{1}{x^{even}}$		0	0
$-\frac{1}{x^{even}}$		0	0

27. $\lim_{x \to -\infty} \left(\frac{1}{2}x - \frac{4}{x^2} \right)$

$= \lim_{x \to -\infty} \frac{1}{2}x - \lim_{x \to -\infty} \frac{4}{x^2}$

$= -\infty - 0 = \boxed{-\infty}$

26. $\lim_{x \to -\infty} \frac{x}{\sqrt{x^2+1}} = \lim_{x \to -\infty} \frac{x}{\sqrt{x^2}} = \lim_{x \to -\infty} \frac{x}{|x|} =$

$= \lim_{x \to -\infty} \frac{x}{-x}$

$= \lim_{x \to -\infty} (-1)$

$= \boxed{-1}$

$\sqrt[n]{x^n} = \begin{cases} x, & n \text{ odd} \\ |x|, & n \text{ even} \end{cases}$

$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

$\sqrt[3]{(-2)^3} = \sqrt[3]{-8} = -2$

$\sqrt{(-2)^2} = \sqrt{4} = 2 = |-2| = -(-2)$

$$\lim_{x\to\infty}\frac{5x-2}{\sqrt{9x^2+3}}=\lim_{x\to\infty}\frac{5x}{\sqrt{9x^2}}=\lim_{x\to\infty}\frac{5x}{|3x|}$$

$$=\lim_{x\to\infty}\frac{5x}{3|x|}=\lim_{x\to\infty}\frac{5x}{3x}=\lim_{x\to\infty}\frac{5}{3}$$

$$=\boxed{\frac{5}{3}}$$

30. $$\lim_{x\to\infty}\frac{x-\cos x}{x}$$

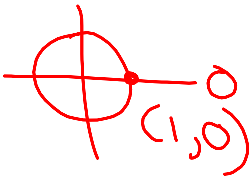
$$=\lim_{x\to\infty}\frac{x}{x}-\lim_{x\to\infty}\frac{\cos x}{x}$$

$$=\lim_{x\to\infty}1-\lim_{x\to\infty}\frac{\overset{\text{bounded}}{[-1,1]}}{x}$$

$$=1-0=\boxed{1}$$

32. $\lim_{x\to\infty} \cos\frac{1}{x} = \cos\left[\lim_{x\to\infty}\frac{1}{x}\right]$

$= \cos 0 = \boxed{1}$



18. c.

$\lim_{x\to\infty} \frac{5x^{3/2}}{4\sqrt{x}+1} \approx \lim_{x\to\infty}\frac{5x^{3/2}}{4x^{1/2}} = \lim_{x\to\infty}\frac{5}{4}x$

$= \boxed{\infty}$

3.7 Optimization Problems

x, y

4. Find two positive numbers whose product is 192 and the sum of the first plus three times the second is a minimum.

$S(x,y) = x + 3y$

sum	is	minimum

$xy = 192$

$x = \frac{192}{y}$

$S(y) = \frac{192}{y} + 3y = 192y^{-1} + 3y$

$S'(y) = -192y^{-2} + 3$

$-\frac{192}{y^2} + 3 = 0$

$3 = \frac{192}{y^2}$

$3y^2 = 192$

$y^2 = 64 \rightarrow y = \pm 8$

$y = 8$

$x = \frac{192}{8} = 24 = x$