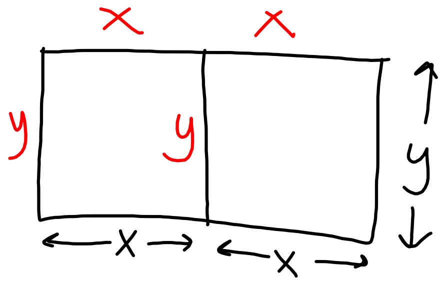
18. A rancher has 200 feet of fencing with which to enclose two adjacent corrals, arranged according to the figure. What dimensions should be used so that the enclosed area will be a maximum?



$A(x,y) = 2xy$

$200 = 4x + 3y$

$200 - 3y = 4x$

$50 - \frac{3}{4}y = x$

$A(y) = 2\left(50 - \frac{3}{4}y\right)y$

$A(y) = 100y - \frac{3}{2}y^2$

$A'(y) = 100 - 3y$

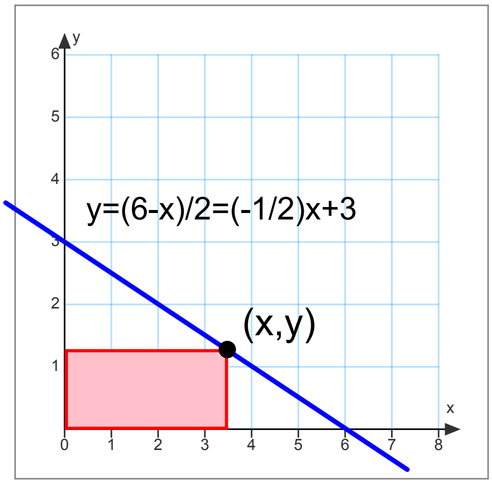
$100 - 3y = 0$

$100 = 3y$

$\boxed{\frac{100}{3}\text{ ft} = y}$

$x = 50 - \frac{\cancel{3}}{\cancel{4}}\left(\frac{\cancel{100}^{25}}{\cancel{3}}\right) = \boxed{25\text{ ft} = x}$

24. A rectangle is bounded by the x- and y-axes and the graph of y=(6-x)/2. What length and width should the rectangle have so that its area is a maximum?



$y = \frac{6-x}{2}$

$y = -\frac{1}{2}x + 3$

$A(x,y) = xy$

$A(x) = x\left(-\frac{1}{2}x + 3\right)$

$A(x) = -\frac{1}{2}x^2 + 3x$

$A'(x) = -x + 3$

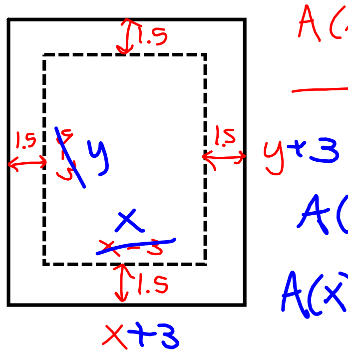
$-x + 3 = 0$

$\boxed{3 = x}$

$y = -\frac{1}{2}(3) + 3$

$\boxed{y = \frac{3}{2}}$

30. A rectangular page is to contain 36 square inches of print. The margins on each side are to be 1.5 inches. Find the dimensions of the page such that the least amount of paper is used.



$A(x,y) = xy$ $\quad (x-3)(y-3) = 36$

$A(x,y) = (x+3)(y+3)$ $\quad xy = 36$

$A(x) = (x+3)\left(\frac{36}{x}+3\right)$ $\quad y = \frac{36}{x}$

$A(x) = 36 + 3x + \frac{3(36)}{x} + 9$

$A(x) = 45 + 3x + 3(36)x^{-1}$

$A'(x) = 3 - 3(36)x^{-2}$

$3 - \frac{3(36)}{x^2} = 0$ $\quad 3 = \frac{3(36)}{x^2}$ $\quad \cancel{3}x^2 = \cancel{3}(36)$

$x = 6$

$$\boxed{\begin{array}{l}\text{width of paper} = 9\text{in} \\ \text{height of paper} = 9\text{in}\end{array}}$$

7.7 Indeterminate Forms & L'Hôpital's Rule

$\frac{0}{0}$, $\frac{\infty}{\infty}$, $0 \cdot \infty$, 1^{∞}, 0^{0}, *and* $\infty - \infty$ are called indeterminate forms.

$\frac{\pm\infty}{\pm\infty}$

L'Hôpital's Rule:

Let f and g be functions that are differentiable on an open interval (a, b) containing c, except possibly at c itself. Assume that $g'(x) \neq 0$ for all x in (a, b), except possibly at c itself. If the limit of f(x)/g(x) as x approaches c produces an indeterminate form $0/0$, ∞/∞, $(-\infty)/\infty$, or $\infty/(-\infty)$, then

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

7.7

12. $\lim_{x\to -1} \frac{x^2-x-2}{x+1} = \lim_{x\to -1} \frac{(x-2)\cancel{(x+1)}}{\cancel{x+1}} = -1-2 = -3$

$\frac{(-1)^2-(-1)-2}{-1+1} = \frac{0}{0}$ ← l'hopital's rule applies

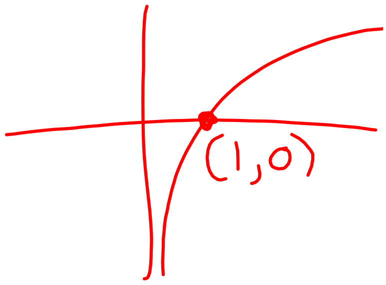
$= \lim_{x\to -1} \frac{2x-1}{1} = 2(-1)-1 = -3$

16. $\lim_{x\to 0^+} \frac{e^x-(1+x)}{x^3} = \frac{e^0-(1+0)}{0^3} = \frac{0}{0}$ l'h applies

$= \lim_{x\to 0^+} \frac{e^x-1}{3x^2} = \frac{0}{0}$ l'h applies

$= \lim_{x\to 0^+} \frac{e^x}{6x} = \frac{1}{0^+} = \boxed{+\infty}$

18. $$\lim_{x\to 1} \frac{\ln x^2}{x^2-1} = \frac{\ln(1)^2}{1^2-1} = \frac{0}{0}$$



$$= \lim_{x\to 1} \frac{\frac{1}{x^2}\cdot \cancel{2x}}{\cancel{2x}} = \boxed{1}$$

20. $$\lim_{x\to 0} \frac{\sin ax}{\sin bx} = \lim_{x\to 0} \frac{\frac{\sin ax}{ax}}{\frac{\sin bx}{bx}} \cdot \frac{b}{a}$$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$= \lim_{x\to 0} \frac{a\cdot\cos ax}{b\cdot \cos bx}$$

$$= \frac{a\cdot\cos 0}{b\cdot\cos 0} = \boxed{\frac{a}{b}}$$