1. $\underbrace{x^2 - 3xy}\ + y^2 = 1$

$x^2 + y^2 = 1 + 3xy$

$2x + 2yy' = 0 + 3y + 3xy'$

$2yy' - 3xy' = 3y - 2x$

$y'(2y - 3x) = 3y - 2x$

$y' = \dfrac{3y - 2x}{2y - 3x} = \dfrac{2x - 3y}{3x - 2y}\Bigg|_{(1,0)} = \boxed{\dfrac{2}{3}}$

$\dfrac{d}{dx}\left[\arcsin(-3x)\right]$ $(\arcsin x)' = \dfrac{1}{\sqrt{1-x^2}}$

$= \dfrac{1}{\sqrt{1-(-3x)^2}} \cdot (-3)$

$= \dfrac{-3}{\sqrt{1-9x^2}}$

$$g(x) = 2x^3 - 21x^2 + 60x$$

$$g'(x) = 6x^2 - 42x + 60$$

$$= 6(x^2 - 7x + 10)$$

$$= 6(x-2)(x-5)$$

$g(0) = 0$ $\quad g(5) = 25$

$g(2) = 52$ $\quad g(6) = 36$

$$f'(x) = \frac{(x+1)^2}{x-4}$$

$f'(-2) \quad f'(0) \quad f'(5)$

$- \quad -1 \quad - \quad 4 \quad +$

$\searrow \quad \searrow \quad \nearrow$

f is increasing on $(4, \infty)$

$$f'(x) = x^4(x-2)(x+3)$$

$f'(-4)$ | $f'(-1)$ | $f'(1)$ | $f'(3)$

$+ \quad -3 \quad - \quad 0 \quad - \quad 2 \quad +$

one relative max @ $x=-3$

$$g(x) = 3x^5 - 30x^4$$

$$g'(x) = 15x^4 - 120x^3$$

$$g''(x) = 60x^3 - 360x^2$$

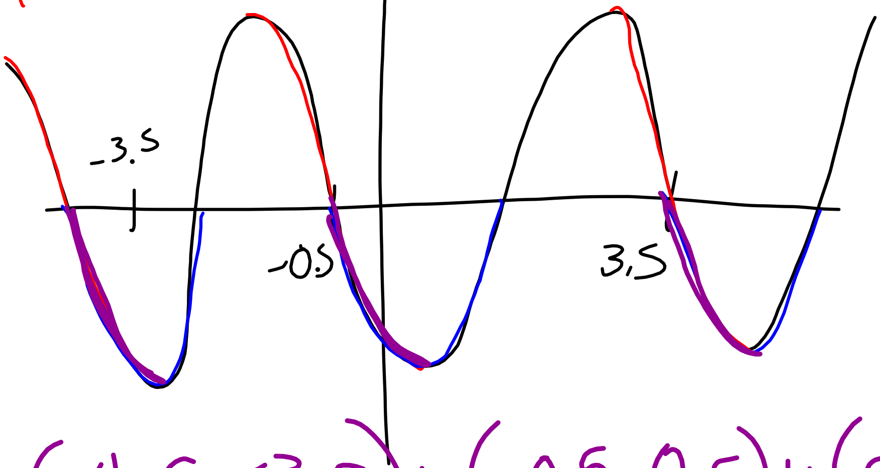
$$= 60x^2(x-6)$$

$f''(-1)$ 0 $f''(1)$ 6 $f''(7)$

$- \quad | \quad - \quad | \quad +$

inflection point @ $x=6$

$f'(x) < 0$ $f''(x) > 0$



$(-4.5, -3.5) \cup (-0.5, 0.5) \cup (3.5, 4.5)$

$$g(x) = -x^4 + 6x^2 - 2x - 3$$

$$g'(x) = -4x^3 + 12x - 2$$

$$g''(x) = -12x^2 + 12 = -12(x^2 - 1)$$

$$= -12(x-1)(x+1)$$

$g''(-2)$ $g''(0)$ $g''(2)$

$-$ $\quad -1 \quad$ $+$ $\quad 1 \quad$ $-$

concave down on

$(-\infty, -1) \cup (1, \infty)$

$$f(x) = \sqrt{2x+1} = (2x+1)^{1/2}$$

$$f'(x) = \frac{1}{\cancel{2}}(2x+1)^{-1/2} \cdot \cancel{2} = \frac{1}{\sqrt{2x+1}}$$

$$\frac{f(12)-f(4)}{12-4} = \frac{\sqrt{2(12)+1} - \sqrt{2(4)+1}}{8}$$

$$= \frac{5-3}{8} = \frac{1}{4}$$

$$\frac{1}{\sqrt{2x+1}} = \frac{1}{4}$$

$$\sqrt{2x+1} = 4$$

$$2x+1 = 16$$

$$2x = 15$$

$$x = 15/2$$

$$V = \frac{1}{3}s^2 h$$

$$\frac{dV}{dt} = \frac{2}{3}s\frac{ds}{dt} \cdot h + \frac{1}{3}s^2 \cdot \frac{dh}{dt}$$

$$= \frac{2}{3}(3)(6)(9) + \frac{1}{3}(3)^2(-1)$$

$$= \boxed{105}$$

7.7 Indeterminate Forms & L'Hôpital's Rule

$\frac{0}{0}$, $\frac{\infty}{\infty}$, $0 \cdot \infty$, 1^{∞}, 0^{0}, *and* $\infty - \infty$ are called indeterminate forms.

L'Hôpital's Rule:

Let f and g be functions that are differentiable on an open interval (a, b) containing c, except possibly at c itself. Assume that $g'(x) \neq 0$ for all x in (a, b), except possibly at c itself. If the limit of f(x)/g(x) as x approaches c produces an indeterminate form 0/0, ∞/∞, (−∞)/∞, or ∞/(−∞), then

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

28. $\lim_{x \to \infty} \frac{x^2}{e^x} = \lim_{x \to \infty} \frac{2x}{e^x} = \lim_{x \to \infty} \frac{2}{e^x} = \boxed{0}$

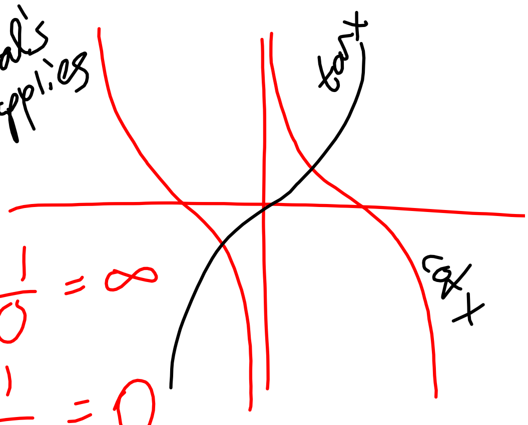
36. $\lim\limits_{x\to\infty} \dfrac{e^{x/2}}{x} = \lim\limits_{x\to\infty} \dfrac{e^{x/2}\cdot\frac{1}{2}}{1} = \infty$

38. $\lim\limits_{x\to 0^+} x^3 \cot x = 0\cdot\infty$

$= \lim\limits_{x\to 0^+} \dfrac{x^3}{\tan x} = \dfrac{0}{0}$ l'hopital's rule applies

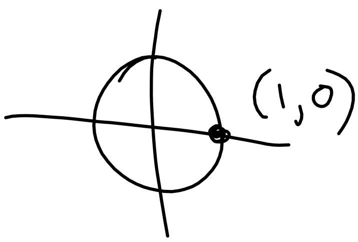
$= \lim\limits_{x\to 0^+} \dfrac{3x^2}{\sec^2 x} = \dfrac{0}{1}$

$= \boxed{0}$



$\dfrac{1}{0} = \infty$

$\dfrac{1}{\infty} = 0$



40. $\lim_{x\to\infty} x\tan\frac{1}{x} = \infty\cdot 0$

$= \lim_{x\to\infty} \frac{\tan\frac{1}{x}}{\frac{1}{x}} = \frac{0}{0}$

$= \lim_{x\to\infty} \frac{\sec^2\left(\frac{1}{x}\right)\cdot \cancel{\frac{-1}{x^2}}}{\cancel{\frac{-1}{x^2}}} = \boxed{1}$

$(x^{-1})' = -1x^{-2}$

$= \frac{-1}{x^2}$

42. $\lim_{x\to 0^+} (e^x + x)^{2/x} \rightarrow (1+0)^\infty = 1^\infty$

$y = \lim_{x\to 0^+} (e^x + x)^{2/x}$

$\ln y = \ln\left[\lim_{x\to 0^+} (e^x+x)^{2/x}\right]$

$\ln y = \lim_{x\to 0^+} \left[\ln(e^x+x)^{2/x}\right]$

$\ln y = \lim_{x\to 0^+} \left[\frac{2}{x}\ln(e^x+x)\right]$

$\infty\cdot 0$

$\ln y = \lim_{x\to 0^+} \left[\frac{2\ln(e^x+x)}{x}\right]$

$\ln y = \lim_{x\to 0^+} \left[\frac{2\cdot\frac{1}{e^x+x}\cdot(e^x+1)}{1}\right]$

$\ln y = 2\cdot\frac{1}{\cancel{e^0}+\cancel{0}}\cdot(\cancel{e^0}+1) = 2\cdot 1\cdot 2 = 4$

$\ln y = 4$

$e^{\ln y} = e^4 \Rightarrow y = \boxed{e^4}$

$\log(a^p) = p\log a$

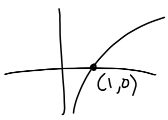
$\log ab = \log a + \log b$

$\log\frac{a}{b} = \log a - \log b$

$\log_a a = 1$

$a^{\log_a x} = x$

$\log_a a^x = x$



$a^{\log_a x} = x$

$e^{\log_e x} = x$

$e^{\ln x} = x$