$$\lim_{x\to 4}\frac{\frac{1}{x}-\frac{1}{4}}{x-4} = \lim_{x\to c}\frac{f(x)-f(c)}{x-c} = f'(c)$$

$$= \left(\frac{1}{x}\right)'\Big|_{x=4} = \boxed{\frac{-1}{16}}$$

$$\lim_{h\to 0}\frac{\sqrt{x+h-3}-\sqrt{x-3}}{h} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = f'(x)$$

$$= \left(\sqrt{x-3}\right)' = \left[(x-3)^{1/2}\right]' = \frac{1}{2}(x-3)^{-1/2} = \boxed{\frac{1}{2\sqrt{x-3}}}$$

$$f(x) = \begin{cases} x^2 - 10 & x \le -1 \\ 2x & \\ 9x & -1 < x < 1 \\ 9 & \\ 6x+3 & x > 1 \\ 6 & \end{cases}$$

$f(-1) = -9 \quad f'(-1) = -2$

$f'(-1) = 9;\ f'(1) = 9$

$f(-1) = -9 \quad f(1) = 9$

$f'(1) = 6$

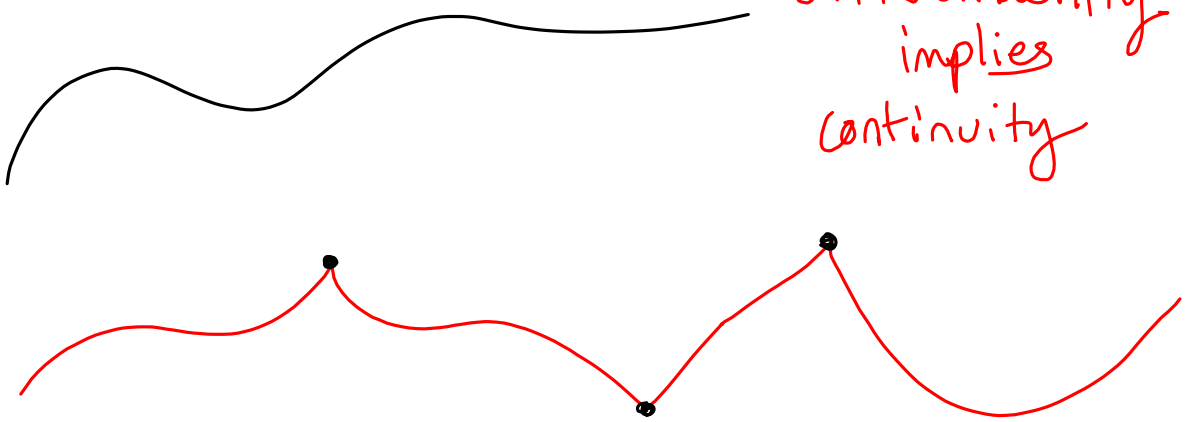
$f(1) = 9$

Is f continuous @ $x = -1$ yes

differentiable @ $x = -1$ no

Continuous @ $x = 1$ no

differentiable @ $x = 1$ no



$$\lim_{x\to\infty} \frac{-5x^7+2x^2-3x+10^{10^{100}}}{\sqrt{169x^{14}-10^{10^{100}}x^2+e}} \leftarrow \text{a googolplex}$$

$$= \lim_{x\to\infty} \frac{-5x^7}{\sqrt{169x^{14}}} = \lim_{x\to\infty} \frac{-5x^7}{|13x^7|} = \lim_{x\to\infty} \frac{-5x^7}{13x^7}$$

$$\sqrt{(13x^7)^2}$$

$$\sqrt[n]{x^n} = \begin{cases} x, & n \text{ odd} \\ |x|, & n \text{ even} \end{cases}$$

$$= \boxed{\frac{-5}{13}}$$

$$\sqrt[3]{(-2)^3} = -2, \quad \sqrt[4]{(-2)^4} = 2$$

$$\lim_{x\to-\infty} \frac{2}{x^5} = \boxed{0}$$

$$\lim_{x\to\infty} \frac{-x^2}{3} = \boxed{-\infty}$$

+ even → − even ↓ + odd → − odd

$$\lim_{x\to0} \frac{\sin x}{x} = 1 \quad ; \quad \lim_{x\to0} \frac{1-\cos x}{x} = 0$$

$$= \lim_{x\to0} \frac{\cos x}{1} = \cos 0 = 1$$

$$\lim_{x\to\frac{\pi}{2}} \frac{\cot x}{x-\frac{\pi}{2}} = \lim_{x\to\frac{\pi}{2}} \frac{-\csc^2 x}{1} = \boxed{-1}$$

$$-\left(\frac{1}{\sin x}\right)^2$$

$$\lim_{x\to\infty} \frac{1}{4x} \cdot e^{x^2} = \lim_{x\to\infty} \frac{e^{x^2}}{4x} \to \frac{\infty}{\infty}$$

$$\downarrow \quad \downarrow$$

$$0 \cdot \infty$$

$$= \lim_{x\to\infty} \frac{e^{x^2} \cdot 2x}{4} = \infty$$