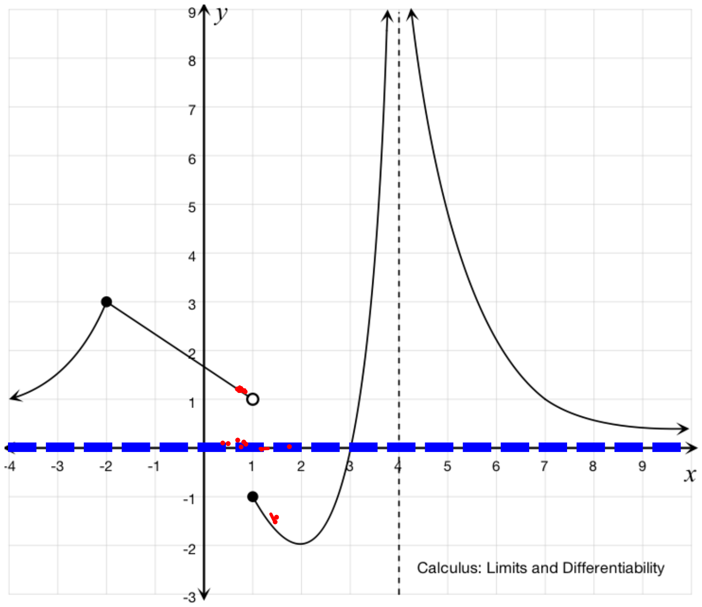
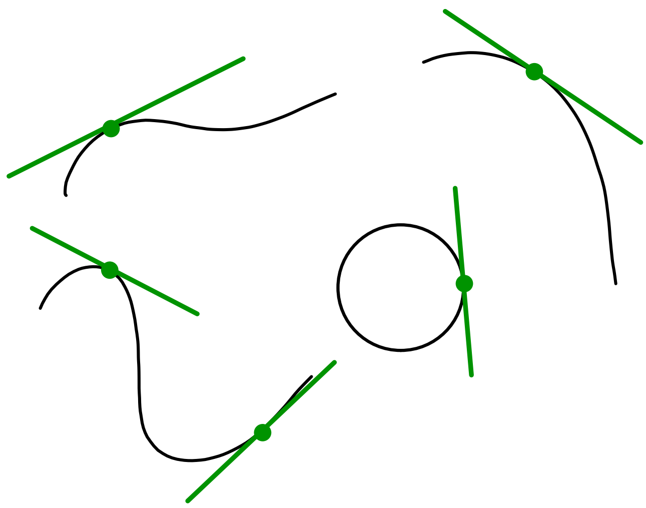
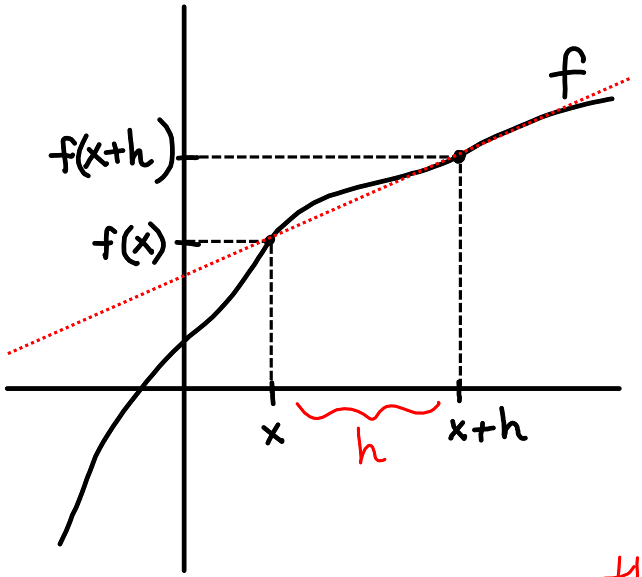


as x approaches...	$f(x)$ approaches...
-2	3
1^- (from the left)	1
1^+ (from the right)	-1
3	0
$-\infty$	0
∞	0
4	∞

tangent lines





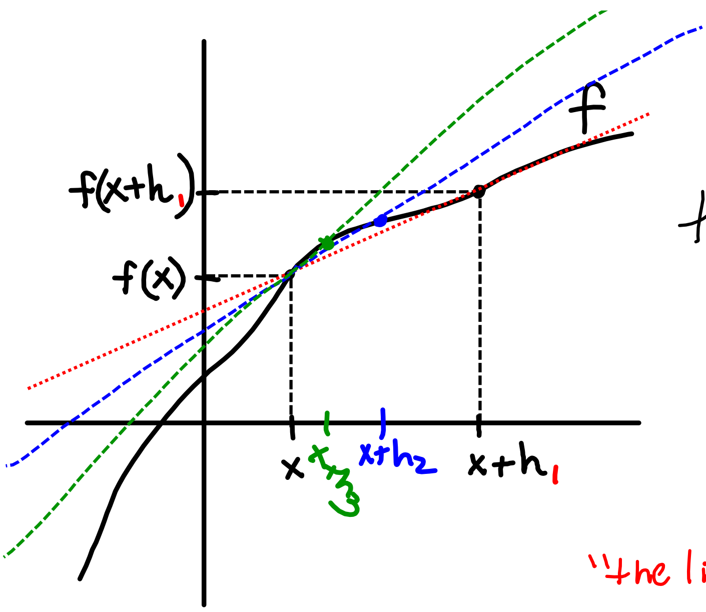
secant line

slope of line through $(x, f(x))$ & $(x+h, f(x+h))$

$$\frac{\Delta y}{\Delta x} = \frac{f(x+h) - f(x)}{x+h-x}$$

$$= \frac{f(x+h) - f(x)}{h}$$

the Difference Quotient



tangent line

as h appoaches 0, the slope of the tangent line at $(x, f(x))$ is

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

"the limit as h appoaches zero of ..."

Δx "delta x"
means change in x

$$\frac{f(x+\Delta x)-f(x)}{\underbrace{\Delta x}} = \frac{f(x+h)-f(x)}{h}$$

treated as a single variable

1.2

$$f(x)=\frac{x-2}{x^2-4}, \quad x\neq 2,-2$$

$$\lim_{x\to 2} f(x) = \frac{1}{4}$$

What happens to *f(x)* as *x* approaches 2?

x	1.9	1.99	1.999	2	2.001	2.01	2.1
f(x)	0.2564	0.2506	0.2501	0.25	0.2499	0.2494	0.2439

Informal Description of the Limit

If *f(x)* becomes arbitrarily close to a single number *L* as *x* approaches *c* from either side, the limit of *f(x)*, as *x* approaches *c*, is *L*.

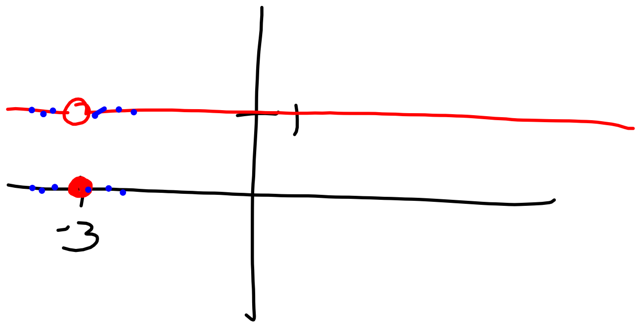
$$\lim_{x \to c} f(x) = L$$

Note: the existence or nonexistence of *f(x)* at *x=c* has no bearing on the existence of the limit as *x* approaches *c*.

A function can be undefined for a certain value of *c* with the limit as *x* approaches *c* still defined.

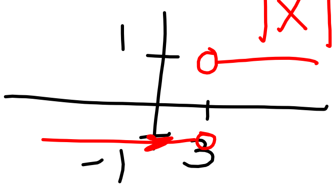
$$\lim_{x \to -3} \frac{\sqrt{1-x}-2}{x+3} = -0.25$$

$$f(x) = \begin{cases} 1, & x \neq -3 \\ 0, & x = -3 \end{cases}$$



$$\lim_{x \to -3} f(x) = 1$$

$$\lim_{x \to 3} \frac{|x-3|}{x-3}$$



$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$\frac{|x-3|}{x-3} = \begin{cases} \frac{x-3}{x-3} = 1, & x-3 > 0 \quad x > 3 \\ \frac{-(x-3)}{x-3} = -1, & x-3 < 0 \quad x < 3 \end{cases}$$

$\lim_{x \to 3^-} f(x) = -1$, $\lim_{x \to 3^+} f(x) = 1$, but

$\lim_{x \to 3} f(x)$ does not exist