## Informal Description of the Limit

If *f(x)* becomes arbitrarily close to a single number *L* as *x* approaches *c* from either side, the limit of *f(x)*, as *x* approaches *c*, is *L*.

$$\lim_{x \to c} f(x) = L$$

Note: the existence or nonexistence of *f(x)* at *x=c* has no bearing on the existence of the limit as *x* approaches *c*.

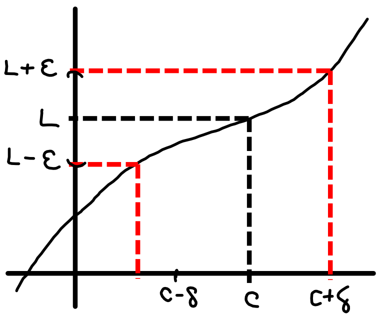
**Building up to the $\epsilon - \delta$ Definition of the Limit**

$\varepsilon$ = epsilon

$\delta$ = delta

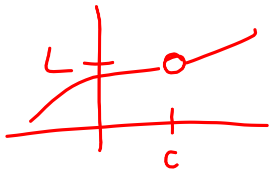
Translating the "informal description": $\lim_{x \to c} f(x) = L$

If $f(x)$ becomes arbitrarily close to a single number $L$ as $x$ approaches $c$ from either side, the limit of $f(x)$, as $x$ approaches $c$, is $L$.




"$f(x)$ becomes arbitrarily close to $L$"

$f(x)$ lies in the interval $(L - \varepsilon, L + \varepsilon)$ for some (really small) $\varepsilon > 0$.




$|f(x) - L| < \varepsilon$

"the distance between $f(x)$ and $L$ is less than $\varepsilon$"

"$x$ approaches $c$"

There exists a (very small) positive number $\delta$ such that $x$ is either in the interval $(c - \delta, c)$ or $(c, c + \delta)$.

$0 < |x - c| < \delta$

The first inequality guarantees that $x \neq c$.

**$\varepsilon - \delta$ Definition of the Limit:**

Let $f$ be a function defined on an open interval containing $c$ (except possibly at $c$) and let $L$ be a real number. The statement

independent arbitrary

$$\lim_{x \to c} f(x) = L$$

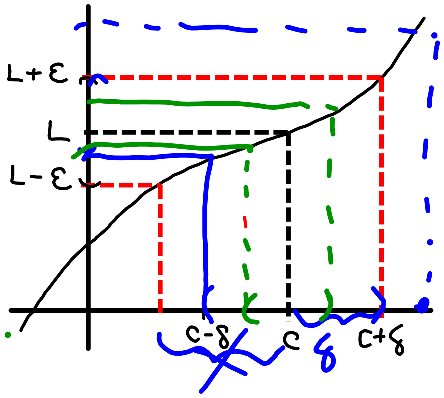
dependent on choice of $\varepsilon$

means that for each $\varepsilon > 0$, there exists a $\delta > 0$ such that if $0 < |x - c| < \delta$, then $|f(x) - L| < \varepsilon$.

If a, then b.

a implies b

$|x-c|<\delta$ implies $|f(x)-L|<\varepsilon$.




**$\varepsilon - \delta$ Definition of the Limit:**

$\lim_{x \to c} f(x) = L$ if given $\varepsilon > 0$, there exists a $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.

$f(x) = 2x - 1$, $c = 4$, $L = 2(4) - 1 = 7$

Find $\lim_{x \to 4} f(x)$ and prove that is the limit using the $\varepsilon$-$\delta$ definition.

Let $\varepsilon > 0$ be given.

We want to find a $\delta > 0$ such that whenever $|x-4| < \delta$, we get that $|f(x) - 7| < \varepsilon$.

$|f(x)-7| = |2x-1-7| = |2x-8| = 2|x-4| < \varepsilon$

$\frac{2|x-4|}{2} < \frac{\varepsilon}{2}$

$|x-4| < \varepsilon/2$

Take $\delta = \varepsilon/2$.

Then whenever $|x-4| < \delta$, we have that

$|f(x) - 7| = 2|x-4| < 2\delta = 2 \cdot \varepsilon/2 = \varepsilon$,

i.e. $|f(x)-7| < \varepsilon$

Hence $\lim_{x \to 4} f(x)$ is indeed 7.

**$\varepsilon - \delta$ Definition of the Limit:**

$\lim_{x \to c} f(x) = L$ if given $\varepsilon > 0$, there exists a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.

$f(x) = -5x+3$ ; find $\lim_{x\to 1} f(x)$ & find a $\delta$.

$5|1-x|$

$5|-x+1|$

$c = 1$

$L = -5(1)+3 = -2$

$|f(x) - L| = |-5x+3-(-2)| = |-5x+5| =$

$= |-5(x-1)| = |-5||x-1| = 5|x-1| < \varepsilon$

$\frac{5|x-1|}{5} < \frac{\varepsilon}{5}$

$\delta = \varepsilon/5$

Prove that the limit is $L$ using the $\varepsilon - \delta$ definition of the limit.

28. $\lim_{x \to -3} (2x + 5) = 2(-3)+5 = -1$

$|f(x) - L| = |2x+5-(-1)| = |2x+6|$

$= 2|x+3|$

$= 2|x-(-3)| < \varepsilon$

$\frac{2|x-(-3)|}{2} < \frac{\varepsilon}{2}$

$\delta = \varepsilon/2$

Take $\delta = \varepsilon/4$

whenever $|x-(-3)| < \delta$, we get that

$|f(x) - (-1)| = 2|x+3| < 2\delta = 2 \cdot \varepsilon/4 = \varepsilon/2 < \varepsilon$

Find $\delta$ for $\varepsilon = 0.01$

24. $\lim_{x \to 4} \left(4 - \frac{x}{2}\right) = 4 - \frac{4}{2} = 2 = L$

$\uparrow$ c

$|f(x) - L| = \left|4 - \frac{x}{2} - 2\right| = \left|-\frac{1}{2}x + 2\right| =$

$= \frac{1}{2}\left|x + \frac{2}{-1/2}\right| = \frac{1}{2}|x - 4| < 0.01$

$\frac{1}{2}|x-4| \,/\, \frac{1}{2} < 0.01 \,/\, \frac{1}{2}$

$|x - 4| < \boxed{0.02 = \delta}$

Classwork: 1.2 #34, 38, 42