Graph the rational function.

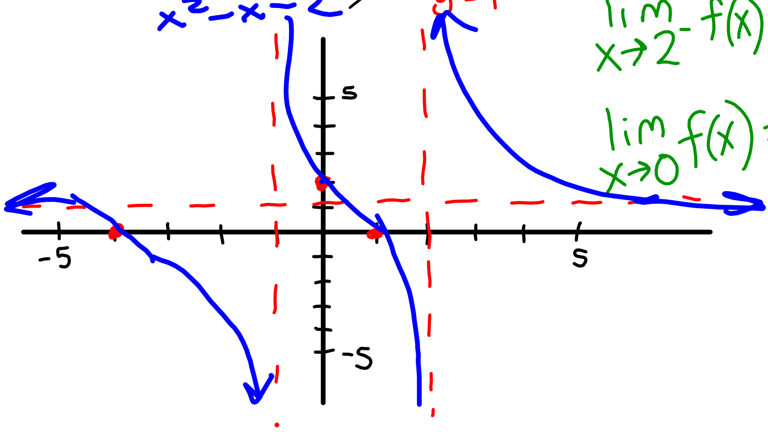
$$f(x) = \frac{\overbrace{(x+4)(x-1)}^{x^2+3x-4}}{\underbrace{(x-2)(x+1)}_{x^2-x-2}} \approx \frac{x^2}{x^2} = 1$$

$y = 1$

$\lim_{x \to \infty} f(x) = 1$

$\lim_{x \to 2^-} f(x) = -\infty$

$\lim_{x \to 0} f(x) = 2$



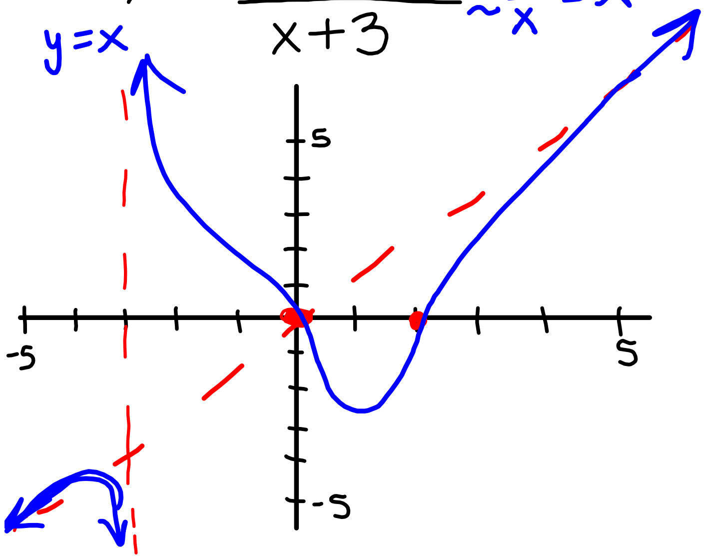
$$f(x) = \frac{x(x-2)}{x+3} \approx \frac{x^2}{x} = x$$

$y = x$

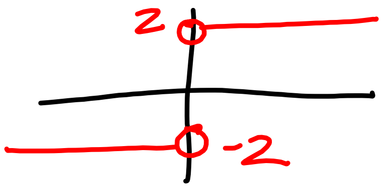
$\lim_{x \to -\infty} f(x) = -\infty$

$\lim_{x \to -3^+} f(x) = \infty$

$\lim_{x \to 2} f(x) = 0$



$$\lim_{x\to 0} \frac{|2x|}{x}$$



$$\lim_{x\to 0^+} f(x) = 2$$

$$\lim_{x\to 0^-} f(x) = -2$$

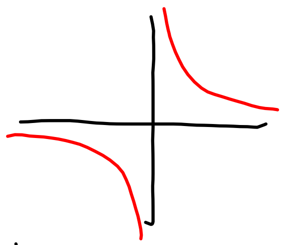
$$\frac{|2x|}{x} = \begin{cases} \frac{2x}{x} = 2, & 2x > 0 \\ & x > 0 \\ \frac{-(2x)}{x} = -2, & 2x < 0 \\ & x < 0 \end{cases}$$

$\lim_{x\to 0} f(x) =$ does not exist

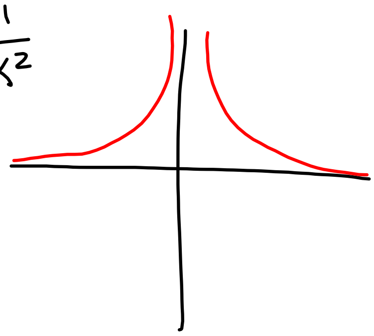
$$|-5| = -(-5) = 5$$

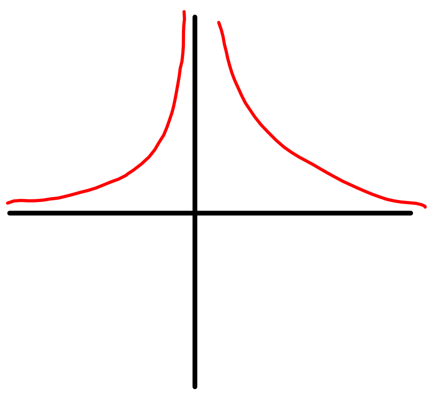
$$\lim_{x\to 0} \frac{1}{x^4} = \boxed{\infty}$$

$\frac{1}{x}$

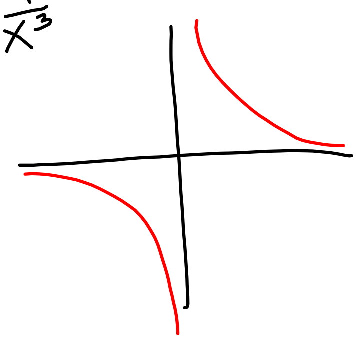


$\frac{1}{x^2}$



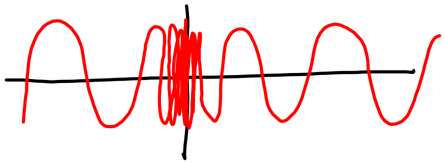


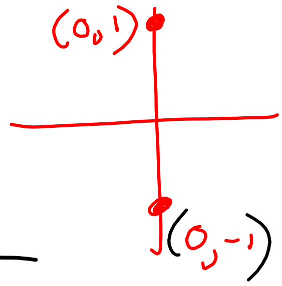
$\frac{1}{x^3}$



$$\lim_{x \to 0} \sin\frac{1}{x}$$

does not exist

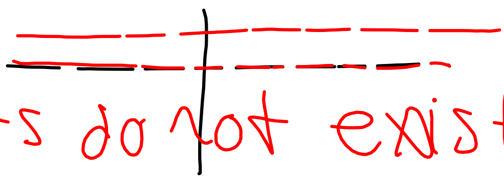




x	$\frac{2}{\pi}$	$\frac{2}{3\pi}$	$\frac{2}{5\pi}$	$\frac{2}{7\pi}$	$\frac{2}{9\pi}$	$\frac{2}{11\pi}$
$\sin\frac{1}{x}$	$\sin\frac{1}{2/\pi} = \sin\frac{\pi}{2} = 1$	$\sin\frac{3\pi}{2} = -1$	$\sin\frac{5\pi}{2} = 1$	-1	1	-1

"Dirichlet Function"

$$f(x) = \begin{cases} 0, & \text{if } x \text{ is rational} \\ 1, & \text{if } x \text{ is irrational} \end{cases}$$



limits do not exist for any X

Informal Description of the Limit

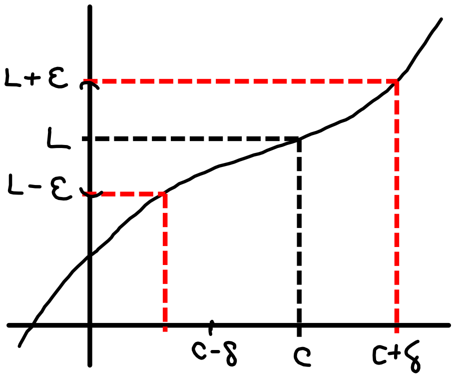
If *f(x)* becomes arbitrarily close to a single number *L* as *x* approaches *c* from either side, the limit of *f(x)*, as *x* approaches *c*, is *L*.

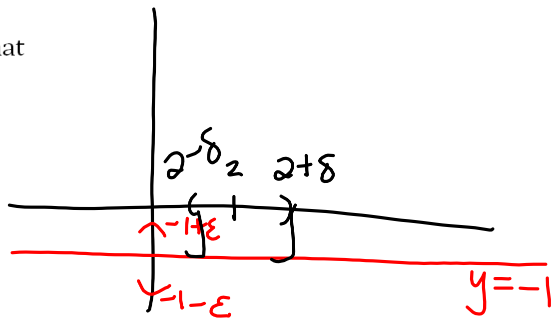
$$\lim_{x \to c} f(x) = L$$

Note: the existence or nonexistence of *f(x)* at *x=c* has no bearing on the existence of the limit as *x* approaches *c*.

$\varepsilon - \delta$ Definition of the Limit:

$\lim_{x\to c} f(x) = L$ if given $\varepsilon > 0$, there exists a $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $0 < |x - c| < \delta$.





Find δ for $\varepsilon = 0.01$

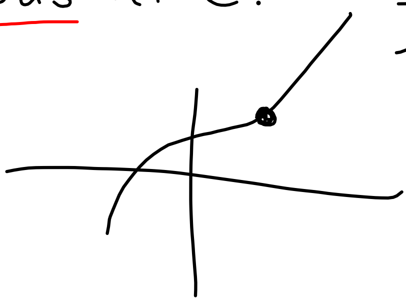
26. $\lim_{x \to 5} (x^2 + 4)$

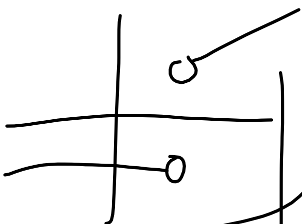
1.3 Evaluating Limits Analytically

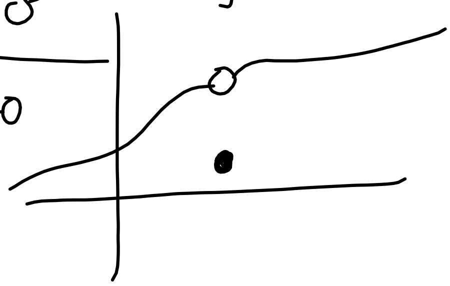
If $\lim_{x \to c} f(x) = f(c)$,

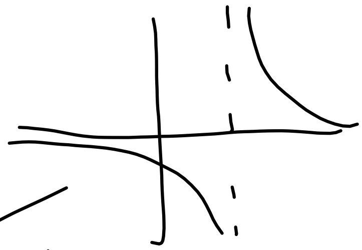
we say that $f(x)$ is

continuous at c.









Evaluating Limits Analytically

Basic Limits

Let $b, c \in \mathbb{R}$, $n > 0$ *an integer*, $f, g -$ *functions*, $\lim_{x\to c} f(x) = L$, $\lim_{x\to c} g(x) = K$

1. Constant $\lim_{x\to c} b = b$
2. Identity $\lim_{x\to c} x = c$
3. Polynomial $\lim_{x\to c} x^n = c^n$
4. Scalar Multiple $\lim_{x\to c}[bf(x)] = bL$
5. Sum or Difference $\lim_{x\to c}[f(x) \pm g(x)] = L \pm K$
6. Product $\lim_{x\to c}[f(x)g(x)] = LK$
7. Quotient $\lim_{x\to c}\left[\frac{f(x)}{g(x)}\right] = \frac{L}{K}$, $K \neq 0$
8. Power $\lim_{x\to c}[f(x)]^n = L^n$

$$\lim_{x\to c}[f(x) \pm g(x)] = \lim_{x\to c} f(x) \pm \lim_{x\to c} g(x)$$

$$\lim_{x\to c}[f(x)g(x)] = \left[\lim_{x\to c} f(x)\right] \cdot \left[\lim_{x\to c} g(x)\right]$$

$$\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}, \quad \lim_{x\to c} g(x) \neq 0$$

$$\lim_{x\to c}[f(x)]^n = \left[\lim_{x\to c} f(x)\right]^n$$

Note: If substitution yields $\frac{0}{0}$, an indeterminate form, the expression must be rewritten in order to evaluate the limit.

$$\lim_{x\to c} a = a$$

$$\lim_{x\to c} x = c$$

$$\lim_{x\to c} x^n = c^n$$

$$\lim_{x\to 5}(-3) = \boxed{-3}$$

$$\lim_{x\to -\pi} x = \boxed{-\pi}$$

$$\lim_{x\to -1} x^5 = (-1)^5 = \boxed{-1}$$

1.3

12. $\lim_{x\to 1}(3x^3-2x^2+4) = 3-2+4 = \boxed{5}$

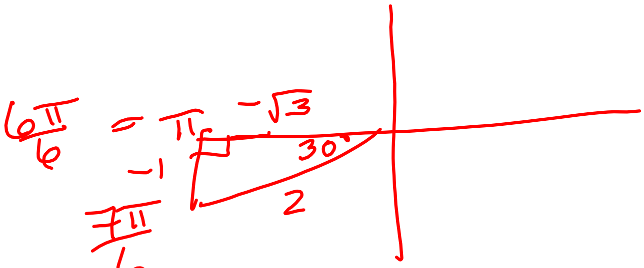
$\sqrt{4}=2$

$x^2=4$

$x=\pm 2$

18. $\lim_{x\to 3}\frac{\sqrt{x+1}}{x-4} = \frac{\sqrt{3+1}}{3-4} = \boxed{-2}$

30. $\lim_{x\to 1}\sin\frac{\pi x}{2} = \sin\frac{\pi}{2} = \boxed{1}$



36. $\lim_{x\to 7}\sec\left(\frac{\pi x}{6}\right) = \sec\frac{7\pi}{6} = \boxed{-\frac{2}{\sqrt{3}}}$

38. $\lim_{x\to c} f(x) = \frac{3}{2}$; $\lim_{x\to c} g(x) = \frac{1}{2}$

(a) $\lim_{x\to c}\left[4f(x)\right] = 4\cdot\lim_{x\to c} f(x) = 4\cdot\frac{3}{2} = \boxed{6}$

(b) $\lim_{x\to c}\left[f(x)+g(x)\right] = \frac{3}{2}+\frac{1}{2} = \boxed{2}$

(c) $\lim_{x\to c}\left[f(x)g(x)\right] = \frac{3}{2}\cdot\frac{1}{2} = \boxed{\frac{3}{4}}$

(d) $\lim_{x\to c}\frac{f(x)}{g(x)} = \frac{3/2}{1/2} = \frac{3}{2}\cdot\frac{2}{1} = \boxed{3}$

$$\lim_{x\to 3}\frac{x^2-2x-3}{x-3} = \lim_{x\to 3}\frac{\cancel{(x-3)}(x+1)}{\cancel{x-3}} = \boxed{4}$$